Reg. No. : ....

Code No. : 10300 E **Sub. Code : AEMA 62**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Sixth Semester

Mathematics

Major Elective — FUZZY MATHEMATICS

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The law of absorption is ————

(a) $A \cup (B \cap C) = B$

(b) $A \cup B = B \cup A$

(c) $A \cup (A \cap B) = A$

(d) ϕ

2. The truth values of fuzzy sets are ———

(a) between 0 or 1 both exclusive

(b) between 0 and 1 both inclusive

(c) either 0 or 1

(d) 0.5

3. Second decomposition theorem states ———

(a) $A = \bigcup_{\alpha \in [0,1]} {}^{\alpha}A$ (b) $A = \bigcup_{\alpha \in [0,1]} {}^{\alpha+}A$

(c) $A = \bigcup_{\alpha \in \wedge(A)} {}^{\alpha}A$ (d) $A = \bigcup_{\alpha \in [0,1]} A$

4. Let A_{α_0} denotes the α-cut of a fuzzy set A at α_0. If $\alpha_1 < \alpha_2$ then

(a) $A_{\alpha_1} \supseteq A_{\alpha_2}$ (b) $A_{\alpha_1} \supset A_{\alpha_2}$

(c) $A_{\alpha_1} \subseteq A_{\alpha_2}$ (d) $A_{\alpha_1} \subset A_{\alpha_2}$

5. $u(a,b) = \min(1, a+b)$ is known as

(a) standard union

(b) algebraic sum

(c) bounded sum

(d) drastic sum

6. $i_w(a,b) =$ ———, $w > 0$.

(a) $1 - \min(1, ((1-a)^w + (1-b)^{1/w}))$

(b) $1 + \min(1, ((1-a)^w + (1-b)^{1/w}))$

(c) $1 - \min(1, ((1-a)^w - (1-b)^{1/w}))$

(d) $1 + \min(1, ((1-a)^w - (1-b)^{1/w}))$

7. A fuzzy number of a fuzzy set A on $\mathbb{R}$ must be

(a) subnormal (b) not convex

(c) convex (d) normal

8. Let A and B be two closed interval such that $A = [a_1, a_2]$ and $B = [b_1, b_2]$ then $B - A$ is

(a) $[b_1 - a_1, b_2 - a_2]$

(b) $[b_1 - a_2, b_2 - a_1]$

(c) $[b_2 - a_1, b_1 - a_2]$

(d) $[b_2 - a_2, b_1 - a_1]$

9. In a linear programming problem, the matrix $A = [a_{ij}], i \in N_m, j \in N_n$ is

(a) goal matrix

(b) consistent matrix

(c) cost matrix

(d) decision matrix

10. $F(x_i, x_j) =$ ———

(a) $\max[f(x_i,x_j)/f(x_j,x_i)]$

(b) $\min[1, f(x_i,x_j)/f(x_j,x_i)]$

(c) $\min[f(x_i,x_j)f(x_j,x_i)]$

(d) $\max[f(x_i,x_j), f(x_j,x_i)]$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) State fundamental properties of crisp set. Among them which are valid in fuzzy set.

Or

(b) Prove that $|A|+|B|=|A\cap B|+|A\cup B|$ is Fuzzy set.

2. (a) State and prove first decomposition theorem.

Or

(b) Let $f: X \to Y$ be an arbitrary crisp function. Then prove that for any $A \in F(X)$, f fuzzified by the extension principle satisfies the equation $f(A) = \bigcup_{\alpha\in[0,1]} f(\alpha + A)$.

13. (a) Let $<i,u,c>$ be a dual triple that satisfies the law of excluded middle and law of contradiction. Then, prove that $<i,u,c>$ does not satisfy the distributive laws.

Or

(b) Assume that a given fuzzy complement c has equilibrium e_c, and fuzzy complement has only one equilibrium point, then prove that $a \le c(a)$ iff $a \le e_c$ and $a \ge c(a)$ iff $a \ge e_c$.

14. (a) If $A(x) = \begin{cases} 0 & \text{if } x \le -1 \,\&\, x > 3 \\ \frac{x+1}{2} & \text{if } -1 < x \le 1 \\ \frac{3-x}{2} & \text{if } 1 < x \le 3 \end{cases}$,

$B(x) = \begin{cases} 0 & \text{if } x \le -1,\ x > 5 \\ \frac{x-1}{2} & \text{if } 1 < x \le 3 \\ \frac{5-x}{2} & \text{if } 3 < x \le 5 \end{cases}$ then find

(i) $\alpha_{(A+B)}$ (ii) $\alpha_{(A-B)}$ (iii) $\alpha_{(A\cdot B)}$ (iv) $\alpha_{(A/B)}$.

Or

(b) Prove that $\min[A, MA\times(B,C)] = \max[\min(A,B), \min(A,c)]$.

15. (a) Explain the method of solving the fuzzy linear programming problem defined by $\max \sum_{j=1}^{n} c_j x_j$ such that $\sum_{j=1}^{n} A_{ij} x_j \leq B_i (i \in N_m)$, $x_j \geq 0 (j \in N_n)$ where B_i and A_{ij} are fuzzy numbers.

Or

(b) Solve the following by graphical method

min $z = x_1 - 2x_2$

subject to $3x_1 - x_2 \geq 1$, $2x_1 + x_2 \leq 6$,

$0 \leq x_2 \leq 2$, $x_1 \geq 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Prove that : A fuzzy set A on $\mathbb{R}$ is convex iff $A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in R$ and all $\lambda \in [0,1]$, where min denotes the minimum operator.

Or

(b) (i) Prove that the law of contradiction and the law of excluded middle are violated for fuzzy sets.

(ii) Prove that the law of absorption is valid.

17. (a) Let $A, B \in F(X)$. Then prove that following properties hold for $\alpha, \beta \in [0,1]$

(i) ${}^{\alpha+}A \subseteq {}^{\alpha}A$

(ii) $\alpha \leq \beta$ implies ${}^{\alpha}A \supseteq {}^{\beta}A$ and ${}^{\alpha+}A \supseteq {}^{\beta+}A$

(iii) ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$ and ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$

(iv) ${}^{\alpha+}(A \cap B) = {}^{\alpha+}A \cap {}^{\alpha+}B$ and ${}^{\alpha+}(A \cup B) = {}^{\alpha+}A \cup {}^{\alpha+}B$

(v) ${}^{\alpha}\overline{A} = {}^{(1-\alpha)+}\overline{A}$.

Or

(b) Let $f : X \to Y$ be an arbitrary crisp function. Then for any $A \in F(X)$ and all $\alpha \in [0,1]$ prove that the following properties of fuzzified by the extension principle hold :

(i) ${}^{\alpha+}[f(A)] = f({}^{\alpha+}A)$

(ii) ${}^{\alpha}[f(A)] \supseteq f({}^{\alpha}A)$.

18. (a) State and prove second characterization theorem of fuzzy complements.

Or

(b) Let i be a t-norm and let $g:[0,1]\rightarrow[0,1]$ be a function such that g is strictly increasing and continuous in (0, 1) and $g(0)=0$, $g(1)=1$. Then prove that the following function i^g defined by $i^g(a,b)=g^{(-1)}(i(g(a),g(b)))$ for all $a,b\in[0,1]$, where $g^{(-1)}$ denotes the pseudo-inverse of i^g is also a t - norm.

19. (a) Let $*\in\{+,-,\cdot,/\}$ and let A,B denote continuous fuzzy numbers. Then prove that the fuzzy set $A*B$ defined by $(A*B)(z)=\sup_{z=x*y}\min[A(x),B(y)]$ $Z\in\mathbb{R}$ is a continuous fuzzy number.

Or

(b) Let MIN and MAX be binary operations on $\mathbb{R}$ defined by

$$MIN(A,B)(z)=\sup_{z=\min(x,y)}\min[A(x),B(y)]$$

$$MAX(A,B)(z)=\sup_{z=\max(x,y)}\min[A(x),B(y)],$$ for all $Z\in\mathbb{R}$. Then for any $A,B,C\in\mathbb{R}$, prove that

(i) $MIN(A,MAX(A\mid B))=A$

(ii) $MIN(A,MIN(B,C))=MIN(MIN(A,B),C)$.

20. (a) Explain linear programming problem.

Or

(b) Solve the following fuzzy linear programming problem $\max z=\cdot 4x_1+\cdot 3x_2$ subject to $x_1+x_2\leq B_1$, $2x_1+x_2\leq B_2$, $x_1,x_2\geq 0$, where B_1 is defined by

$$B_1(x) = \begin{cases} 1 & \text{when} \quad x \le 400 \\ \dfrac{500 - x}{100} & \text{when} \quad 400 < x \le 500 \\ 0 & \text{when} \quad 500 < x \end{cases} \quad \text{and}$$

B_2 is defined.

$$\text{by } B_2(x) = \begin{cases} 1 & \text{when} \quad x \le 500 \\ \dfrac{600 - x}{100} & \text{when} \quad 500 < x \le 600 \\ 0 & \text{when} \quad 600 < x. \end{cases}$$

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Code No. : 10291 E **Sub. Code : AMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fourth Semester

Mathematics – Core

ABSTRACT ALGEBRA

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. In $(\mathbb{Z}_7 - \{0\}, \odot)$, the inverse of 3 is ____________.

   (a) 2 (b) 3

   (c) 4 (d) 5

2. The order of $-i$ in $(c^*, -)$ is ____________.

   (a) 2 (b) infinite

   (c) 1 (d) 4

3. For group $(\mathbb{Z}_{12}, \oplus)$, the number of generators is ____________.

   (a) 4 (b) 3

   (c) 2 (d) 5

4. Choose the correct statement from the following statements.

   (a) Every cyclic group is abelian

   (b) Every abelian group is cyclic

   (c) Every element of a cyclic group is a generator of the group

   (d) $(\mathbb{Q}, +, \cdot)$ is a cyclic group

5. The kernel of the homomorphism $f : (\mathbb{Z}, +) \to \{1, -1\}$ defined by $f(n) = \begin{cases} 1 & if \quad n \text{ is even} \\ -1 & if \quad n \text{ is odd} \end{cases}$, is ____________.

   (a) $2\mathbb{Z}$ (b) $\mathbb{Z}$

   (c) $\{0\}$ (d) $\{1, -1\}$

6. The number of automorphisms of a cyclic group of order 'n' is ____________.

   (a) n (b) $\varphi(n)$

   (c) n^2 (d) 1

7. If R is a commutative ring, then $\forall a, b \in R$, ____________.

   (a) $(a-b)^2 = a^2 - b^2$

   (b) $(a+b)^2 = a^2 + b^2$

   (c) $(a+b)^2 = a^2 + 2ab + b^2$

   (d) $(a+b)^2 \neq 0$

8. An example of an infinite commutative ring without identity is ____________.

   (a) $(\mathbb{Z}, +, \cdot)$ (b) $(\mathbb{Z}_n, \oplus, \otimes)$

   (c) $(2\mathbb{Z}, +, \cdot)$ (d) $M_2(R)$

9. The map $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = x^2 + 3$ is ____________.

   (a) a ring homomorphism

   (b) not a ring homomorphism

   (c) a ring isomorphism

   (d) a group homomorphism

10. Let $f : \mathbf{C} \to \mathbf{C}$ defined by $f(z) = \bar{z}$. Then $Ker f$ is ____________.

    (a) φ (b) $\{0\}$

    (c) $\{1\}$ (d) $\{i\}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let H be a subgroup of a group G. Prove that

    (i) the identity element of H is the same as that of G.

    (ii) for each $a \in H$, the inverse of 'a' in H is the same as the inverse of 'a' in G.

    Or

    (b) Let G be a group and $H = \{a / a \in G$ and $ax = xa \; \forall x \in G\}$. Show that H is a subgroup of G.

12. (a) State and prove Lagrange's theorem.

    Or

    (b) Let H be a subgroup of G. Prove that the number of left cosets of H is the same as the number of right cosets of H.

13. (a) Let M and N be normal subgroups of a group G such that $M \cap N = \{e\}$. Show that every element of M commutes with every element of N.

    Or

    (b) Let $f : G \to G'$ be a homomorphism. Prove that the Kernel K of f is a normal subgroup of G.

[P.T.O.]

14. (a) If R is a ring such that $a^2 = a \forall a \in R$. Prove the following

(i) $a + a = 0$

(ii) $a + b = 0 \Rightarrow a = b$

(iii) $ab = ba$.

Or

(b) Show that $\mathbb{Z}_n$ is an integral domain iff n is prime.

15. (a) Show that $R[x]$ is an integral domain iff R is an integral domain.

Or

(b) If $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = r$ where $x = qn + r$ and $0 \le r \le n$, prove that f is a homomorphism.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let G be the set of all real numbers except -1. Define $*$ on G by $a * b = a + b + ab$. Show that $(G, *)$ is a group.

Or

(b) Let A and B be two subgroup of a group G. Show that AB is a subgroup of G if and only if $AB = BA$.

17. (a) Show that a subgroup of a cyclic group is cyclic.

Or

(b) Prove that a group G has no proper subgroups if it is a cyclic group of prime order.

18. (a) State and prove Cayley's theorem.

Or

(b) Let N be a subgroup of a group G. Prove that the following are equivalent

(i) N is a normal subgroup of G

(ii) $aNa^{-1} = N \ \forall a \in G$

(iii) $aNa^{-1} \le N \ \forall a \in G$

(iv) $ana^{-1} \in N \ \forall n \in N$ and $a \in G$.

19. (a) Let R be a commutative ring with identity. Show that R is a field iff R has no proper ideals.

Or

(b) Let R be a commutative ring with identity. Prove that an ideal M of R is maximal iff R/M is a field.

20. (a) State and prove division algorithm.

Or

(b) State and prove fundamental theorem of homomorphism on rings.

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(7 pages) **Reg. No. :** ....

Code No. : 10292 E **Sub. Code : AMMA 51**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Fifth Semester

Mathematics — Core

LINEAR ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following is a vector space under usual addition and scalar multiplication?

   (a) $V = \{a + b\sqrt{2} + c\sqrt{3} / a,b,c \in Q\}$ over Q

   (b) Z over Q

   (c) $Q[x]$ over R

   (d) Z over Z_5

2. The Kernel of the linear transformation $T : V_3(R) \to V_3(R)$ defined by $T(a,b,c) = (a,b,0)$ is ————

   (a) $\{(0,0,0)\}$ (b) $\{(0,0,c)/c \in R\}$

   (c) $\{(c,0,0)/c \in R\}$ (d) $\{(0,c,0)/c \in R\}$

3. If $S = \{(1,0,0),(2,0,0),(3,0,0)\}$ in $V_3(R)$ then $L(S) =$ ————

   (a) $\{(0,x,0)/x \in R\}$ (b) $\{(0,0,x)/x \in R\}$

   (c) $\{(x,0,0)/x \in R\}$ (d) $\{(0,0,0)\}$

4. $\dim V_n(R) =$ ————

   (a) $\frac{n-1}{2}$ (b) $n+1$

   (c) $n-1$ (d) n

5. The matrix of the linear transformation $T : V_3(R) \to V_2(R)$ given by $T(a,b,c) = (a+b, 2c-a)$ with respect to the standard basis is ————

   (a) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 2 \end{pmatrix}$

   (c) $\begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$

6. The norm of the vector in $V_3(R)$ with standard inner product (1, 2, 3) is ————

   (a) 5 (b) $\sqrt{15}$

   (c) $\sqrt{14}$ (d) $3\sqrt{38}$

7. The inverse of the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ is ————

   (a) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

   (c) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

8. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $|A| =$ ————

   (a) $ad - bc$ (b) $ab - cd$

   (c) $ac - bd$ (d) $ab - dc$

9. The characteristic polynomial of the matrix $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is ————

   (a) x^2 (b) $x^2 - 1$

   (c) $1 - x^2$ (d) $x - 1$

10. The eigen values of $\begin{pmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$ are

    (a) 1, 1, 2 (b) 3, 5, 3

    (c) 3, 4, 1 (d) 3, 0, 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) If A and B are subspaces of V prove that $A + B = \{v \in V / v = a + b, a \in A, b \in B\}$ is a subspace of V. Further show that $A + B$ is the smallest subspace containing A and B.

    Or

    (b) Show that $T : R^2 \to R^2$ defined by $T(a,b) = (2a - 3b, a + 4b)$ is a linear transformation.

12. (a) Prove that $S = \{(1,0,,0),(0,1,0),(1,1,1),(1,1,0)\}$ spans the vector space $V_3(R)$ but is not a basis.

    Or

    (b) Prove that any two bases of a finite dimensional vector space V have the same number of elements.

[P.T.O.]

13. (a) Find the set of all unit vectors in $V_3(R)$ with standard norm.

Or

(b) Let W_1 and W_2 be subspaces of a finite dimensional inner product space. Show that $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$.

14. (a) Find the characteristic equation of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

Or

(b) Show that a square matrix A is involutory iff $A = A^{-1}$.

15. (a) Let f be the bilinear form defined on $V_2(R)$ by $f(x,y) = x_1y_1 + x_2y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of f with respect to the basis $\{(1,1), (1,2)\}$.

Or

(b) State and prove Cayley Hamilton theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Let V be a vector space over F and W a subspace of V. Let $V/W = \{W + V / v \in V\}$. Show that V/W is a vector space over F under the following operations.

(i) $(W + V_1)(W + V_2) = W + V_1 + V_2$

(ii) $\alpha(W + V_1) = W + \alpha V_1$.

Or

(b) Let V be a vector space over a field F. Let A and B be subspaces of V. Show that $\frac{A+B}{A} \cong \frac{B}{A \cap B}$.

17. (a) Prove that any vector space of dimension n over a field F is isomorphic to $V_n(F)$.

Or

(b) Let V be a vectors space over a field F and S be a non-empty subset of V prove that

(i) $L(S)$ is a subspace of V

(ii) $S \subseteq L(S)$

(iii) If W is any subspace of V such that $S \subseteq W$, then $L(S) \subseteq W$ (i.e). $L(S)$ is the smallest subspace of V containing S.

18. (a) Let V be a finite dimensional inner product space. Let W be a subspace of V. Prove that $V = W \oplus W^{\perp}$.

Or

(b) Apply Gram - Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{V_1, V_2, V_3\}$ where $V_1 = (1,0,1)$, $V_2 = (1,3,1)$, $V_3 = (3,2,1)$.

19. (a) P.T. any square matrix A can be uniquely expressed as the sum of a Hermitian matrix and a skew Hermitian matrix.

Or

(b) Find the inverse of the matrix $\begin{bmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ using Cayley - Hamilton theorem.

20. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

Or

(b) Reduce the quadratic form $x_1^2 + 4x_1x_2 + 4x_1x_3 + 4x_2^2 + 16x_2x_3 + 4x_3^2$ to the diagonal form using Lagrange's method.

(6 pages) **Reg. No. :** ....

Code No. : 10293 E **Sub. Code : AMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fifth Semester

Mathematics — Core

REAL ANALYSIS

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If M is a discrete metric space then $B(a, 4)$ is

   (a) $\{a\}$ (b) M

   (c) ϕ (d) 4

2. Z is

   (a) not open in R (b) open in R

   (c) not closed (d) an interval

3. Which is not correct?

   (a) $A \subset \overline{A}$ (b) $Int\, A \subset A$

   (c) $Int\, A \subset \overline{A}$ (d) $\overline{A} \subset \text{int}\, A$

4. $D(Q) =$ ________.

   (a) Q (b) ϕ

   (c) R (d) Z

5. If $f : R \to R$ is continuous at a then $w(f, a)$ is

   (a) 0 (b) a

   (c) 1 (d) ∞

6. If $f : R \to R$ which is uniformly continuous?

   (a) $f(x) = x^2$ (b) $f(x) = \sin x$

   (c) $f(x) = \frac{1}{x}$ (d) none

7. Choose the correct statement

   (a) R is connected

   (b) Q is connected

   (c) A subspace of a connected space is connected

   (d) Union of two connected subsets of a metric space is connected

8. $\{(-n, n)/n \in N\}$ is an open cover for ________.

   (a) R (b) N

   (c) Z (d) Q

9. In $\mathbb{R}$, (5, 6) is a

   (a) Closed Set (b) Compact

   (c) Not a compact set (d) None

10. Any continuous function $f : [a, b] \to \mathbb{R}$ is

    (a) onto (b) 1-1

    (c) open (d) not onto

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing (a) or (b).

11. (a) Define discrete metric d. Prove that d is a metric on M.

    Or

    (b) Let (M, d) be a metric space. Let $A, B \subseteq M$. Then prove the following :

    (i) $A \subseteq B \Rightarrow Int\, A \subseteq Int\, B$

    (ii) $Int(A \cap B) = Int\, A \cap Int\, B$.

12. (a) Let (M, d) be a metric space. Prove that any convergent sequence in M is a Cauchy Sequence.

    Or

    (b) Prove that any discrete metric space is complete.

13. (a) Define Uniformly Continuous Function. Prove that the function $f : (0, 1) \to \mathbb{R}$ define by $f(x) = \frac{1}{x}$ is not uniformly continuous.

    Or

    (b) Prove : $f : \mathbb{R} \to \mathbb{R}$ is continuous at $a \in R$ $\Leftrightarrow w(f, a) = 0$.

14. (a) Prove that any continuous image of a connected set is connected.

    Or

    (b) Define a connected set. Prove that any discrete metric space M with more than one point is disconnected.

15. (a) Prove that any compact subset A of a metric space M is bounded.

Or

(b) Define totally bounded metric space. Prove that any compact metric space is totally bounded.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) For $p \geq 1$, let l_p denote the set of all sequences (x_n) such that $\sum_{n=1}^{\infty} |x_n|^p$ is convergent. Define $d(x, y) = \left\{ \sum_{n=1}^{\infty} (x_n - y_n)^p \right\}^{\frac{1}{p}}$ where $x = (x_n)$ and $y = (y_n)$. Prove that d is a metric on l_p.

Or

(b) Let (M, d) be a metric space. Let x, y be two distinct points of M. Prove that there exists two disjoint open balls with centres x and y respectively.

17. (a) Prove that $\mathbb{R}^n$ with usual metric is complete.

Or

(b) State and prove Cantor's intersection theorem.

18. (a) If (M_1, d_1) and (M_2, d_2) are two metric spaces and $a \in M_1$ then prove that $f : M_1 \to M_2$ is continuous at a $\Leftrightarrow (x_n) \to a \Rightarrow (f(x_n)) \to f(a)$.

Or

(b) Let $f : [a, b] \to \mathbb{R}$ be a monotonic function. Prove that set of points of $[a, b]$ at which f is discontinuous is countable.

19. (a) Let A be a connected subset of M, $A \subseteq B \subseteq \overline{A}$. Show that $\overline{A}$ is connected.

Or

(b) Prove : A subspace of $\mathbb{R}$ is connected $\Leftrightarrow$ is is an interval.

20. (a) State and prove Heine Borel Theorem.

Or

(b) Prove : A metric space (M, d) is totally bounded $\Leftrightarrow$ every sequence in M has a Cauchy Subsequence.

(6 pages)

Reg. No. : ....

Code No. : 10295 E **Sub. Code : AMMA 62**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The Independence number of K_p is ————

(a) 2 (b) p

(c) $p-1$ (d) 1

2. $K_{m,n}$ = ————

(a) $K_m + K_n$ (b) $K_m \cup K_n$

(c) $\overline{K}_m + \overline{K}_n$ (d) $K_m \cap K_n$

3. Petersen graph is ———— graph

(a) Eulerian

(b) Hamiltonian

(c) Eulerian, Non-Hamiltonian

(d) Non Eulerian, Non-Hamiltonian

4. If e is a bridge of a graph G, then $w(G-e)=$ ————

(a) $w(G)$ (b) $w(G)+1$

(c) $w(G)-1$ (d) $w(G)-2$

5. (A) Every edge of a tree is a bridge

(B) A block has no cut vertex

(a) (A), (B) are correct

(b) (A) is correct, (B) is wrong

(c) (A) is wrong, (B) is correct

(d) (A), (B) are wrong

6. The chromatic number of a tree is ————

(a) 1 (b) 2

(c) 4 (d) 0

7. The number of faces in K_4 is ————

(a) 4 (b) 2

(c) 6 (d) 5

8. The thickness of K_9, $\theta(K_9)=$ ————

(a) 1 (b) 2

(c) 3 (d) 4

9. A digraph on p vertices is functional if the out degree of every vertex is ————

(a) 1 (b) 2

(c) p (d) $p-1$

10. (A) Strongly connected ⇒ weakly connected

(B) Weakly connected ⇒ unilaterally connected

(a) (A), (B) are correct

(b) (A) is correct, (B) is wrong

(c) (A) is wrong, (B) is correct

(d) (A), (B) are wrong

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Show that sum of degrees of vertices of a graph is twice the number of edges.

Or

(b) If G is a (p, q) graph, show that $\delta \le \frac{2q}{p} \le \Delta$.

12. (a) If $\delta \ge K$ show that the graph G has a path of length K.

Or

(b) Show that a closed walk of odd length contains a cycle.

13. (a) Prove that every connected graph has a spanning tree.

Or

(b) Prove that G is a tree iff G is connected and every line of G is a bridge.

14. (a) Prove that K_5 is non-planar.

Or

(b) Prove that every planar graph G with $p \ge 3$ vertices has at least 3 points of degree less than 6.

[P.T.O.]

15. (a) In a digraph D, show that the sum of indegrees of all the vertices equals sum of their out degrees, each being equal to the number of arcs.

Or

(b) If two diagraphs are isomorphic, prove that corresponding points have the same degree pair.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Show that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

Or

(b) (i) Prove that, every graph′ is an intersection graph.

(ii) A (p, q) graph has t vertices of degree m and all other vertices are of degree n. Show that $(m-n)t+pn=2q$.

17. (a) Prove that an edge x of a connected graph G is a bridge iff x is not on any cycle of G.

Or

(b) State and prove the necessary and sufficient condition for a partition of an even number to be graphical.

18. (a) Prove that a (p, q) graph G is a tree iff G is acyclic and $p=q+1$.

Or

(b) If G is a plane (p, q) graph in which every face is an n – cycle, prove that $q=\frac{n(p-2)}{n-2}$.

19. (a) State and prove Dirac's theorem.

Or

(b) Prove that every uniquely n – colourable graph is $(n-1)$ connected.

20. (a) Prove that a weak digraph D is Eulerian iff every vertex of D has equal indegree and out degree.

Or

(b) Find the chromatic polynomial of the graph with partition $(3, 3, 3, 3, 2)$.

(7 pages) Reg. No. : ....

Code No. : 10296 E **Sub. Code : AMMA 63**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics — Core

NUMBER THEORY

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If a and b are any positive integers, then there exist a positive n such that $n\,a \geq b$ by
   (a) Well ordering principle
   (b) Archimedean property
   (c) Finite Induction principle
   (d) Binomial theorem

2. For $n \geq 1$, $\frac{1.3.5....(4n-1)}{[1.3.5....(2n-1)]^2}\binom{2n}{n}$.
   (a) $\binom{4n}{2n}$
   (b) $\binom{4n}{n}$
   (c) $\binom{2n}{n}$
   (d) $\binom{4n}{3n}$

3. Match for integers a, b, c

(i)	$a \mid 1$	(1)	$a \mid c$
(ii)	$a \mid b$ and $b \mid a$	(2)	$a = \pm 1$
(iii)	$a \mid b$ and $c \mid d$	(3)	$a = \pm b$
(iv)	$a \mid b$ and $b \mid c$	(4)	$ac \mid bd$

   (a) (i) – 2, (ii) – 3, (iii) – 1, (iv) – 4
   (b) (i) – 2, (ii) – 3, (iii) – 4, (iv) – 1
   (c) (i) – 1, (ii) – 2, (iii) – 3, (iv) – 4
   (d) (i) – 2, (ii) – 4, (iii) – 3, (iv) – 1

4. If a and b are integers, with $b \neq 0$, then there exist unique integers q and r such that $a = qb + r$ where the value of r is
   (a) $0 \leq r \leq |b|$ (b) $0 \leq r \leq |a|$
   (c) $0 \leq r < |b|$ (d) $0 < r < |b|$

5. If p is a prime and $p \mid ab$, then ————
   (a) $p \mid a$ and $p \mid b$ (b) $p \nmid a$ and $p \nmid b$
   (c) $p \mid a$ or $p \mid b$ (d) none of these

6. There is an infinite number of primes of the form ————
   (a) $4n$ (b) $4n+1$
   (c) $4n+2$ (d) $4n+3$

7. Which of the following is true?
   (i) $-56 \equiv 9 \pmod 7$
   (ii) $-11 \equiv 9 \pmod 7$
   (a) (i) alone
   (b) (ii) alone
   (c) (i) and (ii) both true
   (d) both false

8. If 'a' is a solution of $p(x) \equiv 0 \pmod n$ and $a \equiv b \pmod n$, then ————
   (a) b is also a solution
   (b) b need not be a solution
   (c) b is sometime a solution
   (d) the value of b is undetermined

9. By Fermat's method factorize 119143 which is ————
   (a) $352^2 - 69^2$
   (b) $(352+69)(352-69)$
   (c) 421 .283
   (d) All the above

10. If p is a prime, then ———— for integer a
   (a) $a^p \equiv 1 \pmod p$
   (b) $a^p \equiv a \pmod p$
   (c) $a^p \equiv 0 \pmod p$
   (d) $a^p \not\equiv a \pmod p$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) State and prove Archimedean property.

Or

(b) Prove by mathematical induction

$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(2n+1)(n+1)}{6}$.

12. (a) If $K > 0$, prove that $gcd(Ka, Kb) = K gcd(a, b)$.

Or

(b) Find the $gcd(12378, 3054)$.

13. (a) Prove that $\sqrt{2}$ is irrational.

Or

(b) If the $n>2$ terms of the arithmetic progression $P, P+d, P+2d, ... P+(n-1)d$ n are all prime numbers, then the common difference d is divisible by every prime 'q' $q<n$.

14. (a) If $ca \equiv cb \pmod{n}$, prove that $a \equiv b \pmod{n/d}$, where $d = gcd(c, n)$.

Or

(b) The linear congruence $ax \equiv b \pmod{n}$ has a solution if and only if $d|b$, where $d = gcd(a, n)$. If $d|b$, prove that if has d mutually in congruent solutions modulo n.

15. (a) State and prove Fermat's little theorem.

Or

(b) Illustrate Fermat's method by finding factor of 119143.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove Binomial theorem.

Or

(b) Illustrate a proof of second principle of finite induction for lucas sequence 1, 3,4, 7, 11, 18, 29, 47, 76...

17. (a) State and prove Division algorithm.

Or

(b) State and prove Euclidean Algorithm.

18. (a) There are an infinite number of primes.

Or

(b) State and prove fundamental theorem of Arithmetic.

19. (a) State and prove Chinese remainder theorem.

Or

(b) Let $n>0$ be fixed and a, b, c, d be arbitrary integers then prove the following properties.

(i) $a \equiv b \pmod{n}$, then $b \equiv a \pmod{n}$

(ii) $a \equiv a \pmod{n}$

(iii) If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$ then $a \equiv c \pmod{n}$

(iv) If $a \equiv b \pmod{n}$, then $a^k \equiv b^k \pmod{n}$.

20. (a) State and prove Wilson theorem.

Or

(b) If P is a prime, prove that $a^p \equiv a \pmod{p}$ for any integer a.

———————

(8 pages) **Reg. No. :** ....

Code No. : 10422 E **Sub. Code : CAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

First/Third Semester

Mathematics – Allied

ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. The least degree of an equation with real coefficients, two of whose roots are $1+i$ and $\sqrt{3}$ is

   (a) 2 (b) 3

   (c) 4 (d) 6

2. If 'a' is a root of $x^4 - 2x^3 + 6x^2 + 2x - 1 = 0$, then ________

   (a) $-a$ is also a root (b) $2a$ is also a root

   (c) $1/a$ is also a root (d) a^2 is also a root

3. One root of $x^3 - x - 3 = 0$ lies between ________

   (a) 0 and 1 (b) 1 and 2

   (c) 0 and –1 (d) –1 and –2

4. When the roots of the equation $3x^3 - 10x^2 + 9x + 2 = 0$ are multiplied by 3, the transformed equation is ________

   (a) $3x^3 - 100x^2 + 900x + 2000 = 0$

   (b) $27x^3 - 90x^2 + 27x + 2 = 0$

   (c) $3x^3 - 30x^2 + 81x + 54 = 0$

   (d) $x^3 - \frac{10}{3}x^2 + 3x + 2/3 = 0$

5. The characteristics equation of $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ is ________

   (a) $x^2 - 2x - 5 = 0$ (b) $x^2 + 2x + 5 - 0$

   (c) $-x^2 - 2x + 5 = 0$ (d) $-x^2 - 2x - 5 = 0$

6. The eigen values of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ are ________

   (a) $-1, 1$ (b) $-1, -1$

   (c) $1, -1$ (d) $1, 1$

7. The complementary function of $(D^3 - 3D^2 + 3D - 1)$ $y = x^3$ is ——————

(a) $e^x(a + bx + cx^2)$

(b) $e^{-x}(a\cos x + b\sin x + c)$

(c) $ae^x + be^{-x} + ce^{2x}$

(d) $e^{-x}(a + bx + cx^2)$

8. The partial differential equation by eliminating the arbitrary constants a and b form $z = axy + b$ is ——————

(a) $px + qy = 0$ (b) $py + qx = 0$

(c) $px - qy = 0$ (d) $py - qx = 0$

9. $L(\sqrt{x}) =$ ——————

(a) $\frac{\sqrt{\pi}}{2s^{3/2}}$ (b) $\frac{1}{s^2}$

(c) $\frac{1}{\sqrt{s}}$ (d) $\frac{\pi}{2s^{3/2}}$

10. $L^{-1}[F(s+a)] =$ ——————

(a) $e^{ax}L^{-1}[F(s)]$ (b) $e^{-ax}L^{-1}[f(s)]$

(c) $e^{ax}L[f(s)]$ (d) $1/a\, F\left(\frac{s}{a}\right)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in arithmetic progression.

Or

(b) Solve $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

12. (a) Increase the roots of equation $4x^5 - 2x^3 + 7x - 3 = 0$ by 2.

Or

(b) Apply Newton's method to obtain the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2.

13. (a) Find the eigen values of the matrices $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ and $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$.

Or

(b) Verify Cayley Hamilton's theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

14. (a) Solve

(i) $(D^2+D+1)^2 y = 0$

(ii) $(D^2+D+1)y = \sin 2x$

Or

(b) Focus on the method of eliminating constants a, b, c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ and form a partial differential equation.

15. (a) Apply Laplace transform to the functions

(i) $t^2 + \cos 2t \ \cos t + \sin^2 2t$

(ii) e^{at}

Or

(b) Find the inverse Laplace transform of

(i) $\frac{1}{(s+3)^2+25}$ and

(ii) $\frac{s}{(s+2)^2}$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4+px^3+qx^2+rx+s=0$ find

(i) $\sum\left(\frac{1}{\alpha}\right)$

(ii) $\sum\left(\frac{\alpha}{\beta}\right)$

(iii) $\sum\left(\frac{1}{\alpha\beta}\right)$

(iv) $\sum \alpha^2\beta$

(v) $\sum \alpha^3$

Or

(b) Find the roots of $3x^6+x^5-27x^4+27x^2-x-3=0$

17. (a) Apply Horner's method to find the positive root of $x^3-x-3=0$ correct to two places of decimals.

Or

(b) Show that $x^3+3x-1=0$ has only one real root and calculate it correct to two places of decimals.

18. (a) Using Cayley Hamilton theorem, find the inverse of the matrix $\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$.

Or

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

19. (a) Solve :

(i) $(D^3 - 3D^2 + 3D - 1)y = x^2 e^x$

(ii) $(D^2 + 5D + 6)y = x^2$

Or

(b) Solve: $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$.

20. (a) Applying Laplace transformation, find

(i) $L(x^2 \cosh ax)$

(ii) $L\left(\frac{1 - \cos x}{x}\right)$

Or

(b) Applying inverse Laplace transformation, find

(i) $L^{-1}\left(\frac{1 + 2s}{(s+2)^2 (s-1)^2}\right)$

(ii) $L^{-1}\left(\frac{s^2 - s + 2}{s(s-3)(s+2)}\right)$

(7 pages) **Reg. No. :** ....

Code No. : 10424 E **Sub. Code : CAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second/Fourth Semester

Mathematic — Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla \cdot \bar{r} =$ __________.

   (a) 1 (b) 0

   (c) 3 (d) $x^2 + y^2 + z^2$

2. A vector function $\bar{f}$ is called solenoidal if __________.

   (a) $div\bar{f} = 0$ (b) $grad\bar{f} = 0$

   (c) $div\bar{f} = \bar{0}$ (d) $curl\bar{f} = 0$

3. $\int_0^1 \int_0^1 dydx =$ __________.

   (a) 2 (b) 1

   (c) 0.5 (d) None of the above

4. $\int_0^a \int_0^b \int_0^c dxdydz =$ __________.

   (a) $a + b + c$ (b) $a^3 + b^3 + c^3$

   (c) abc (d) $(abc)^3$

5. If C is the straight line joining (0, 0, 0) and (1, 1, 1) then $\int_c \bar{r} \cdot d\bar{r}$ is __________.

   (a) $\frac{1}{2}$ (b) 1

   (c) $\frac{3}{2}$ (d) 2

6. Value of $\int (xdy - ydx)$ around the circle $x^2 + y^2 = 1$ is __________.

   (a) 0 (b) $\frac{\pi}{2}$

   (c) π (d) 2π

7. Value of $\iint_S \hat{n} \times \overline{F} ds$ is ________.

(a) $\iiint_V div \overline{F} dV$ (b) $\iiint_V Curl \overline{F} dV$

(c) $\iiint_V \overline{F} dV$ (d) zero

8. If S is the sphere $x^2 + y^2 + z^2 = 1$ the value of $\iint_S \overline{r} \cdot \hat{n} ds$ is ________.

(a) $\frac{4}{3}\pi$ (b) 3π

(c) 4π (d) 2π

9. If $f(x)$ is an odd function then $\int_{-a}^{a} f(x) dx$ = ________.

(a) $2\int_0^a f(x) dx$ (b) 0

(c) $\frac{2}{\pi}\int_0^a f(x) dx$ (d) None of these

10. What is the period of the periodic function $\sin nx$?

(a) π (b) 2π

(c) $2n\pi$ (d) $\frac{2\pi}{n}$

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\nabla\phi = yz\overline{i} + zx\overline{j} + xy\overline{k}$ find $\phi(x, y, z)$.

Or

(b) Prove that $\overline{f} = (x^2 - yz)\overline{i} + (y - zx)\overline{j} + (z^2 - xy)\overline{k}$ is irrotational.

12. (a) Evaluate the following integral $\int_0^1 \int_0^2 (x+2) dy dx$.

Or

(b) Evaluate the integral $\int_0^1 dx \int_0^2 dy \int_1^2 x^2 yz dz$.

13. (a) If $\overline{f} = (2y+3)\overline{i} + xz\overline{j} + (yz - x)\overline{k}$ evaluate $\int_C \overline{f} \cdot d\overline{r}$ along the path C the straight line joining (0, 0, 0) and (2, 1, 1).

Or

(b) Evaluate $\iint_S \overline{F} \cdot \hat{n} ds$ where $\overline{F} = (x + 2y^2)\overline{i} - 2x\overline{j} + 2yz\overline{k}$ where S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

14. (a) Calculate $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the square by lines $x = 0$, $x = 1$, $y = 0$, $y = 1$.

Or

(b) Using Stoke's theorem calculate $\int_C (yzdx + zxdy + xydz)$ where C is the any closed curve.

15. (a) Find the Fourier series for the function $f(x) = x^2$ where $-\pi \le x \le \pi$.

Or

(b) Find the Fourier sine series for the function $f(x) = \pi - x$ in the interval $(0, \pi)$.

SECTION C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Find the unit normal at (6, 4, 3) to $xy + yz + zx = 54$.

Or

(b) Prove that $\overline{F} = 3y^4z^2\overline{i} + 4x^3z^2\overline{j} - 3x^2y^2\overline{k}$ is solenoidal.

17. (a) Estimate $\int_2^3 \int_1^2 \frac{dxdy}{xy}$.

Or

(b) Estimate $I = \int_0^{\log a} \int_0^x \int_0^{x+y} e^{x+y+z} dzdydx$.

18. (a) If $\overline{f} = (3x^2 + 6y)\overline{i} - 14yz\overline{j} + 20xz^2\overline{k}$ determine $\int_C \overline{f} \cdot d\overline{r}$ from (0, 0, 0) to (1, 1, 1) along the curve $x = t$, $y = t^2$, $z = t^3$.

Or

(b) Determine $\iint \overline{f} \cdot \hat{n} ds$ where $\overline{f} = (x^3 - yz)\overline{i} - 2x^2y\overline{j} + 2\overline{k}$ and S is the surface of the cube bounded by $x = 0$, $y = 0$, $z = 0$, $x = a$, $y = a$ and $z = a$.

19. (a) Verify Gauss divergence theorem for $\overline{f} = y\overline{i} + x\overline{j} + z^2\overline{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2$, $z = 0$ and $z = k$.

Or

(b) Verify Stoke's theorem for $\overline{f} = (2x - y)\overline{i} - yz^2\overline{j} - y^2z\overline{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

20. (a) If $f(x) = -x$ in $-\pi < x < 0$

x in $0 \leq x < \pi$

express $f(x)$ as Fourier Series in the interval $-\pi$ to π. Deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.$

Or

(b) Formulate a cosine-series in the range 0 to π for

$$f(x) = x \quad \left(0 < x < \frac{\pi}{2}\right)$$

$$= \pi - x \quad \left(\frac{\pi}{2} < x < \pi\right)$$

(7 pages) **Reg. No. :** ....

Code No. : 10425 E **Sub. Code : CAST 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second/Fourth Semester

Mathematics — Allied

STATISTICS — II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Geometric Mean of Paasche and Laspeyre Index Numbers is ________.
   (a) Bowley Index Number
   (b) Fisher Index Number
   (c) Marshal Edgeworth Index Number
   (d) Kelly Index Number

2. Aggregate expenditure method of cost of living index is nothing but the following ________.
   (a) Marshall Edgeworth Index
   (b) Laspeyere's Index
   (c) Fisher's Index
   (d) Bowley's Index

3. Type II error is otherwise known as ________.
   (a) Rejection error (b) Acceptance error
   (c) Probable error (d) Standard error

4. Sample is a part of ________.
   (a) Sampling (b) Population
   (c) Probability (d) None of these

5. Test for equality of means based on two small samples is based on ________.
   (a) Normal distribution
   (b) χ^2 distribution
   (c) F-distribution
   (d) t-distribution

6. In which one of the following sampling design proportional allocation is used?

(a) SRS

(b) Stratified random sample

(c) Systematic sample

(d) None

7. The error degrees of freedom for two-way classified data is ________.

(a) $pq(n-1)$ (b) $np(q-1)$

(c) $nq(p-1)$ (d) $(p-1)(q-1)$

8. The basic principles of design of experiment are ________.

(a) Local control (b) Randomization

(c) Replication (d) All of these

9. Which of the following is suitable for P-chart?

(a) Number of defective pieces

(b) Measurable values

(c) Number of defects in a unit

(d) None of the above

10. In SQC, the important tools is ________.

(a) Control charts (b) Sampling plans

(c) Both (a) and (b) (d) None of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define Time Reversal Test and Factor Reversal Test.

Or

(b) From the following data, construct the index number taking 1987 as base.

Years	1987	1988	1989	1990	1991	1992
Price of rice per kg	5.00	6.00	6.50	7.00	7.50	8.00

12. (a) Explain (i) Critical Region (ii) Level of Significance.

Or

(b) A coin is tossed 144 times and a person gets 80 heads. Can we say that the coin is unbiased one?

13. (a) Explain any two test of significance based on t-distribution.

Or

(b) Explain the test of independence of two attributes in a $m \times n$ contingency table.

14. (a) Define one-way classification and two-way classification.

Or

(b) Explain RBD.

15. (a) Define control chart.

Or

(b) Point out Seven Quality Control Tools.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Calculate (i) Laspeyere's (ii) Paasches (iii) Fisher's Index numbers of the following data given below. Hence or otherwise find Edgeworth and Bowley's Index Numbers.

Commodities	Base Year 1990		Current Year 1992	
	Price	Quantity	Price	Quantity
A	2	10	3	12
B	5	16	6.5	11
C	3.5	18	4	16
D	7	21	9	25
E	3	11	3.5	20

Or

(b) Prove that Fishers Index Number is an ideal index number.

17. (a) Two populations have their means equal but the standard deviation σ of one is twice the other. (i) Show that in the sample of size 2,000 from each drawn under simple sampling conditions the difference of means will in all probability not exceed 0.15 σ. where σ is the smaller SD. (ii) Find the probability that the difference will exceed half this amount.

Or

(b) A Machine put out 16 imperfect articles in a sample of 500 articles. After the machine is overhauled it puts out 3 defective articles in a sample of 100. Has the machine improved?

18. (a) Two random samples gave the following results.

Sample	Size	Sample Mean	Sum of squares of deviations from the mean
I	10	15	90
II	12	14	108

Test whether the sample could have come from the same normal population.

Or

(b) A group of 10 rats fed on a diet A and another group of 8 rats fed on a different diet B recorded the following increases in weights in gms.

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	1	10	2	8	–	–

Test whether diet A is superior to diet B.

19. (a) Explain the analysis of Latin Square Design (LSD).

Or

(b) Analyse the one-way ANOVA for the following.

Batches	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
A	1600	1610	1650	1680	1700	1720	1800	–
B	1580	1640	1640	1700	1750	–	–	–
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680	–	–

20. (a) What are the ways Sampling Inspection can be carried out? Explain.

Or

(b) Describe the construction of P-chart for fixed and variable sample sizes.

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Code No. : 10418 E **Sub. Code : CMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

First Semester

Mathematics — Core

CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The locus of the limiting position of the intersecting points of any two curves of a family of curves is called ———— of the family.

   (a) involute (b) evolute

   (c) curvature (d) envelope

2. The curvature of the circle is ———— of its radius.

   (a) half (b) square

   (c) reciprocal (d) square root

3. $\Gamma(n+1)$ = ————

   (a) $(n-1)\Gamma(n)$ (b) $(n+1)!$

   (c) $(n+1)\Gamma(n)$ (d) $n!$

4. $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ = ————

   (a) π (b) $\sqrt{\pi}$

   (c) $\frac{\pi}{2}$ (d) $\frac{\sqrt{\pi}}{2}$

5. $\int_0^4\int_0^4 dx\,dy$ = ————

   (a) 4 (b) 16

   (c) 12 (d) 8

6. $\int_0^a\int_0^a\int_0^a a^2\,dx\,dy\,dz =$ ______

(a) a^3 (b) a^4

(c) a^5 (d) a^2

7. If $f(a)$ and $f(b)$ have like signs, ______ roots of $f(x)=0$ lie between a and b.

(a) even number of

(b) no

(c) odd number of

(d) either (a) or (b)

8. The sum of roots of the equation $ax^4+4bx^3+6cx^2+4dx+e=0$ is ______

(a) $-4b$ (b) $\frac{4b}{a}$

(c) $\frac{-4b}{a}$ (d) $4b$

9. The coefficients of an odd degree reciprocal equation have all like signs. Then ______ is its root.

(a) -1 (b) 1

(c) ± 1 (d) 0

10. The number of positive roots of the equation $x^5-6x^2-4x+5=0$ is ______

(a) atleast two (b) atmost one

(c) atleast one (d) atmost two

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the radius of curvature at any point of the catenary $y = c\cosh\frac{x}{c}$ is equal to the length of the portion of the normal intercepted between the curve and the axis of x.

Or

(b) Develop an equation of a curve which forms the envelope of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{k^2-a^2}=1$ where 'a' is the parameter.

12. (a) Change the order of integration in $\int_0^a\int_{\frac{x^2}{a}}^{2a-x} xy\,dx\,dy$ and evaluate it.

Or

(b) Using Jacobians, evaluate $\iint_R (x-y)^4 e^{x+y}\,dx\,dy$ where R is the square with vertices (1, 0), (2, 1), (1, 2) and (0, 1).

[P.T.O.]

13. (a) Evaluate :

(i) $\int_0^{\infty} e^{-x^2} dx$

(ii) $\int_0^{\frac{\pi}{2}} \sin^{10}\theta \, d\theta$.

Or

(b) Evaluate the integral $\iint x^p y^q \, dy \, dx$ over the triangle $x > 0$, $y > 0$, $x + y < 1$ in terms of Gamma functions.

14. (a) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression.

Or

(b) If $a + b + c + d = 0$, show that

$$\frac{a^5 + b^5 + c^5 + d^5}{5} = \frac{a^2 + b^2 + c^2 + d^2}{2} \cdot \frac{a^3 + b^3 + c^3 + d^3}{3}.$$

15. (a) Increase by 7, the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.

Or

(b) Show that the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$ has atleast four imaginary roots.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Or

(b) (i) Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at the point (2, 1).

(ii) Prove that the radius of curvature at any point of cycloid $x = a(\theta + \sin\theta)$; $y = a(1 - \cos\theta)$ is $4a \cos\frac{\theta}{2}$.

17. (a) Evaluate $\iiint xyz \, dx \, dy \, dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Use the substitution $x + y + z = u$, $y + z = uv$, $z = uvw$ to evaluate the integral $\iiint [xyz(1 - x - y - z)]^{\frac{1}{2}} \, dx \, dy \, dz$ taken over the tetrahedral volume enclosed by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

18. (a) Establish relation between Beta and Gamma functions.

Or

(b) Evaluate in terms of Gamma functions, the integral $\iiint x^{p} y^{q} z^{r}\, dx\, dy\, dz$ taken over the volume of the tetrahedron given by $x \geq 0$, $y \geq 0$, $z \geq 0$ and $x + y + z \leq 1$.

19. (a) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in Geometric progression and hence solve $27x^3 + 42x^2 - 28x - 8 = 0$, whose roots are in geometric progression.

Or

(b) Show that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1 = 0$ is zero.

20. (a) Solve the equation :

$$6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0.$$

Or

(b) Solve the equation :
$x^4 + 20x^3 - 143x^2 + 430x + 462 = 0$ by removing its second term.

(7 pages)

Reg. No. : ....

Code No. : 10419 E **Sub. Code : CMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics — Core

DIFFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The general solution of the differential equation $y - p(x+1) = p$ is

(a) $y = p(x+2)$ (b) $y = cx$

(c) $y = cx + 2c$ (d) $y = c(x+1)$

2. $\frac{1}{D^2 + a^2}\cos ax =$ ________.

(a) $\frac{x}{2a}\sin ax$ (b) $\frac{-x}{2a}\sin ax$

(c) $\frac{x}{2}\sin ax$ (d) $\frac{x}{a}\sin ax$

3. The differential equation with constant coefficients obtained from $x^2\frac{d^2y}{dx^2} + y = 3x^2$ by substituting $x = e^z$, $D = \frac{d}{dz}$ is

(a) $(D^2 - D + 1)y = 3x^2$ (b) $(D^2 + D - 1)y = 3z^2$

(c) $(D^2 - D + 1)y = 3z^2$ (d) $(D^2 - D + 1)y = 3e^{2z}$

4. The complementary function of $(x^2D^2 + xD + 1)y = \log x$ is

(a) $A + Bx$

(b) $A\cos(\log x) + B\sin(\log x)$

(c) $A + B$

(d) $(A + Bx)e^x$

5. The middle point of the line joining the points (1, 2, 8) and (1, 1, 3) is ________.

(a) (1, 3, 11) (b) $\left(1, \frac{3}{2}, \frac{11}{2}\right)$

(c) $\left(1, \frac{2}{3}, \frac{11}{2}\right)$ (d) $\left(1, \frac{2}{3}, \frac{2}{11}\right)$

6. The angle between the planes $2x+4y-6z=1$ and $3x+6y-5z+4=0$ is ________.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) None of the above

7. A straight line is equally inclined to the three coordinate axes. Then that angle = ________.

(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\frac{1}{3}\right)$

(c) $\cos^{-1}\left(\frac{1}{2}\right)$ (d) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

8. On which plane does the line $\frac{x-4}{2}=\frac{y-2}{3}=\frac{z-3}{6}$ lie?

(a) $4x+3y+20z=5$ (b) $4x+2y+3z=2$

(c) $3x-4y+z=7$ (d) $2x-2y+z=1$

9. The radius of the sphere $2x^2+2y^2+2z^2-2x+2y-4z-5=0$ is ________.

(a) 2 (b) $\sqrt{\pi}$

(c) 1 (d) $\frac{\sqrt{\pi}}{2}$

10. The equation of the tangent plane of the sphere $x^2+y^2+z^2=9$ at (1, –2, 2) is

(a) $x-2y+2z+9=0$ (b) $x-2y+2z-9=0$

(c) $x+2y+2z+9=0$ (d) $x-2y-2z-9=0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve : $y=xp+x\left(1+p^2\right)^{\frac{1}{2}}$.

Or

(b) Sovle : $tdx=(t-2x)dt$

$tdy=(tx+ty+2x-t)dt$.

12. (a) Solve : $\left(D^3-3D^2+3D-1\right)y=x^2e^x$.

Or

(b) Solve : $x^2y''+3xy'+y=\frac{1}{(1-x)^2}$.

13. (a) Show that the points (2, 5, –4), (1, 4, –3), (4, 7, –6) and (5, 8, –7) are the vertices of a parallelogram.

Or

(b) Prove that the lines $\frac{x-3}{2}=\frac{y-2}{-5}=\frac{z-1}{3}$ and $\frac{x-1}{-4}=y+2=\frac{z-6}{2}$ are coplanar.

14. (a) Find the distance between the parallel planes $2x - 3y + 6z + 12 = 0$, $2x - 3y + 6z - 2 = 0$

Or

(b) Find the equation of the image of the line $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-4}{2}$ in the plane $2x - y + z + 3 = 0$.

15. (a) Find the equation of the sphere which has its center at the point (6, –1, 2) and touches the plane $2x - y + 2z - 2 = 0$.

Or

(b) Show that the plane $2x + y - 2z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0$. Find the point of contact.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve : $\frac{dx}{dt} + 2x - 3y = t$

$\frac{dy}{dt} - 3x + 2y = e^{2t}$.

Or

(b) Solve : $(px - y)(x + yp) = a^2 p$ (Take $x^2 = X$, $y^2 = Y$).

17. (a) Solve : $(D^2 - 2D + 4)y = e^x \cos x$.

Or

(b) Solve the differential equation $\frac{d^2y}{dx^2} + n^2 y = \cos nx$.

18. (a) Show that the lines whose direction cosines are related as $3l + 4m + 5n = 0$, $l^2 + m^2 - n^2 = 0$ are parallel.

Or

(b) A moving plane passes through a fixed point (α, β, γ) and intersects the coordinate axes at A, B, C. Show that the locus of centroid of the triangle ABC is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.

19. (a) Find the coordinates of the foot of the perpendicular drawn from the point (2, 3, 1) to the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

Or

(b) Show that the lines $\frac{x-2}{1} = \frac{y-4}{+2} = \frac{z-5}{2}$ and $\frac{x-5}{+2} = y - 8 = \frac{z-7}{2}$ are coplanar. Find the point of intersection. Also, find the equation of the plane determined by the lines.

20. (a) A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Show that the locus of the center of the sphere $OABC$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

Or

(b) A sphere of constant radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$.

(6 pages) Reg. No. : ....

Code No. : 10420 E **Sub. Code : CMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Third Semester

Mathematics — Core

SEQUENCES AND SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The g.l.b. of the sequence $1, \frac{1}{2}, \frac{1}{3}, \cdots \frac{1}{n}, \cdots$ is ———

   (a) 1

   (b) 0

   (c) ∞

   (d) $\frac{1}{2}$

2. $\lim_{n \to \infty} \frac{1}{n^2} =$ ———

   (a) 0 (b) ∞

   (c) 1 (d) $\frac{1}{n}$

3. $\lim_{n \to \infty} \left(n^{1/n}\right) =$ ———

   (a) 0 (b) ∞

   (c) 1 (d) $\frac{1}{n}$

4. $\lim_{n \to \infty} \frac{3n-4}{2n+7} =$ ———

   (a) $\frac{3}{2}$ (b) $\frac{2}{3}$

   (c) $\frac{4}{7}$ (d) $\frac{-4}{7}$

5. The series $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$

   (a) converges to zero

   (b) diverges to ∞

   (c) converges to one

   (d) diverges to $-\infty$

6. $\sum \frac{1}{4n^2 - 1} =$ ———

   (a) $\frac{1}{2}$ (b) $\frac{1}{4}$

   (c) -1 (d) 0

7. The series $\sum \frac{1}{(\log n)^n}$ is ———

   (a) divergent (b) diverges to ∞

   (c) convergent (d) none

8. The series $\Sigma \frac{x^n}{n}$ converges if ———

   (a) $x = 1$ (b) $x > 1$

   (c) $x = 0$ (d) $x < 1$

9. $\sum (-1)^n \left(1 + \frac{1}{n}\right)$ ———

   (a) is oscillating (b) diverges

   (c) converges (d) none

10. The series $\sum (-1)^n \sin\left(\frac{1}{n}\right)$ ———

    (a) diverges (b) converges

    (c) is constant (d) converges to $\frac{1}{n}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) State and prove Cauchy - Schwarz inequality.

    Or

    (b) Define bounded sequence. Give an example.

12. (a) Show that $\left((-1)^n\right)$ is not a convergent sequence.

    Or

    (b) Show that if $(a_n) \to a$ and $K \in \mathbb{R}$ then $(Ka_n) \to Ka$.

13. (a) Show that $\lim_{n \to \infty} \frac{n!}{n^n} = 0$.

    Or

    (b) Prove that any Cauchy sequence is a bounded sequence.

14. (a) Test the convergence of the series $\frac{1}{3}x + \frac{1}{3} \cdot \frac{2}{5} x^2 + \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{3}{7} x^3 + \cdots$

    Or

    (b) Test the convergence of $\sum \frac{n^3 + a}{2^n + a}$.

[P.T.O.]

15. (a) Show that the series $\sum \frac{(-1)^{n+1}}{\log(n+1)}$ converges.

Or

(a) Prove that $\sum_{n=2}^{\infty}\left(\frac{\sin n}{\log n}\right)$ is convergent.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) For any two real numbers x and y, prove

(i) $|x+y| \leq |x| + |y|$

(ii) $|x-y| \geq \left||x| - |y|\right|$.

Or

(b) (i) If a,b,c are positive real numbers such that $a^2+b^2+c^2=27$ then show that $a^3+b^3+c^3 \geq 81$.

(ii) Define monotonic sequences. Give an for each example.

17. (a) (i) Prove that a sequence cannot converge to two different limits.

(ii) Prove if $(a_n) \to a$ and $a_n \geq 0$ for all n then $a \geq 0$.

Or

(b) Discuss the behaviour of the geometric sequence (r^n).

18. (a) State and prove Cauchy's first limit theorem.

Or

(b) State and prove comparison test.

19. (a) State and prove Kummer's test.

Or

(b) State and prove Gauss's test.

20. (a) State and prove Dirichlet's test.

Or

(b) State and prove Abel's test.

(6 pages)

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Code No. : 10421 E **Sub. Code : CMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fourth Semester

Mathematics – Core

ABSTRACT ALGEBRA

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In the group $(z_8, \oplus)$, the order of 2 is ________.

(a) 0 (b) 1

(c) 3 (d) 4

2. In the group $\{1, w, w^2\}$, $w^3 = 1$, under usual multiplication, the identity element is ________.

(a) 0 (b) 1

(c) 2 (d) 3

3. The generator of the cyclic group $(z_8, \oplus)$ is ________.

(a) 0 (b) 2

(c) 3 (d) 6

4. If H is a subgroup of G, then $a \in H \Rightarrow aH =$ ________.

(a) H (b) a

(c) φ (d) G

5. In the quotient group G/N, N is ________.

(a) a subgroup of G

(b) a cyclic sub group of G

(c) a normal sub group of G

(d) an abelian sub group of G

6. The product of two even permutation is ________ permulation.

(a) Cycle (b) Odd

(c) Even (d) Odd and even

7. In the ring $(Z, +, \cdot)$, units are ________.

(a) 0 (b) 1

(c) -1 (d) $1, -1$

8. In the ring $(Q, +, \cdot)$, identity element is ________.

(a) 0 (b) 1

(c) 2 (d) 3

9. If $f : R \to R'$ be an ring isomorphism then $f(-a) =$ ________ $\forall a \in R$.

(a) $-f(a)$ (b) $-a$

(c) $f(a)$ (d) a

10. If $f : R \to R'$ is $1-1$, then $Kref =$ ________.

(a) R (b) R'

(c) $0'$ (d) $\{0\}$

PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) In R^*, $a * b = \dfrac{ab}{2}$ then prove that $(R^*, *)$ is a group.

Or

(b) Let G be a group and $a \in G$. Let $H_a = \{x \mid x \in G$ and $ax = xa\}$. Prove that H_a is a subgroup of G.

12. (a) Prove that a subgroup of cyclic group is cyclic.

Or

(b) State and prove Euler's theorem.

13. (a) Show that if a group G has exactly one subgroup H of given order, then prove that H is a normal subgroup of G.

Or

(b) Let $f : G \to G'$ be an isomorphism. Let $a \in G$. Then prove that order of a is equal to the order of $f(a)$.

14. (a) Show that every field is an integral domain.

Or

(b) Prove that any finite integral domain is a field.

15. (a) Let R and R' be rings and $f : R \to R'$ be a homomorphism. Then S is an ideal of R, prove that $f(S)$ is an ideal of R'.

Or

(b) Let $f : R \to R'$ be a homomorphism. Prove that Kerf is an ideal of R'.

[P.T.O.]

PART C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Let A and B be two subgroups of a group G. Then prove that AB is a subgroup of G iff $AB = BA$.

Or

(b) If H, K are two finite subgroups of a group G, then show that $|HK| = \frac{|H||K|}{|H \cap K|}$.

17. (a) Let H be a subgroup of a group G. Then prove that

(i) $aH = bH \Rightarrow a^{-1}b \in H$

(ii) $a \in bH \Rightarrow a^{-1} \in Hb^{-1}$

(iii) $a \in bH \Rightarrow aH = bH$.

Or

(b) State and prove Lagrange's theorem.

18. (a) State and prove Cayley's theorem.

Or

(b) If a permutation $p \in s_n$ is a product of r transposition and also a product of s transpositions, then prove that either r and s are both even or both odd.

19. (a) Let R be a commutative ring with identity. Prove that a_n ideal M of R is maximal iff R/M is a field.

Or

(b) Prove that Z_n is an integral domain ⇔ n is prime.

20. (a) State and prove fundamental theorem of homomorphism.

Or

(b) Prove that the only isomorphism $f : Q \rightarrow Q$ is the identity map.

(7 pages)

Reg. No. : ....

Code No. : 10428 E **Sub. Code : CNMA 31**

U.G. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics — Non Major Elective

MATHEMATICS FOR COMPETITIVE EXAMINATION – I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

Choose the correct answer :

1. $\frac{11}{4} = \frac{77}{?}$

(a) 28 (b) $\frac{77}{28}$

(c) 44 (d) 308

2. $5005 - 5000 \div 10.00 =$ __________.

(a) 0.5 (b) 50

(c) 5000 (d) 4505

3. Divide 455 in the ratio 4 : 3.

(a) 260, 195 (b) 255, 200

(c) 250, 205 (d) 265, 190

4. If $2A = 3B = 4C$ then $A : B : C$ is

(a) $2:3:4$ (b) $4:3:2$

(c) $6:4:3$ (d) $3:4:6$

5. A, B, C started a business by investing Rs.1,20,000, Rs.1,35,000 and Rs.1,50,000 respectively. A's share out of an annual profit of Rs.56,700 is __________.

(a) Rs.15,000

(b) Rs.16,800

(c) Rs.17,800

(d) Rs.20,000

6. A number increased by $37\frac{1}{2}\%$ gives 33. The number is __________.

(a) 22 (b) 24

(c) 25 (d) 27

7. By selling an article for Rs.100, one loses Rs.10. Then loss percentage is __________.

(a) $11\frac{1}{9}\%$ (b) $9\frac{1}{11}\%$

(c) 10% (d) 11%

8. Cost price = Rs.56.25, Profit = 20%, selling price = __________.

(a) Rs.62.50 (b) Rs.60

(c) Rs.67.50 (d) Rs.66.25

9. $\frac{3}{4}$ of a number is 19 less than the original number. The number is __________.

(a) 84 (b) 64

(c) 76 (d) 72

10. 11 times a number gives 132. The number is __________.

(a) 11 (b) 12

(c) 13.2 (d) 13

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) $15\frac{2}{3} \times 3\frac{1}{6} + 6\frac{1}{3} = 11\frac{7}{18} + ?$

Or

(b) Find the value of $\frac{9^2 \times 18^4}{3^{16}}$.

12. (a) In a mixture of 35 litres, the ratio of milk and water is 4:1. Now, 7 litres of water is added to the mixture. Find the ratio of milk and water in the new mixture.

Or

(b) Find the fourth proportional to 4, 5 and 12.

13. (a) A and B started a business and invested Rs.20,000 and Rs.25,000 respectively. After 4 months B left and C joined by investing Rs.15,000. At the end of the year, there was a profit of Rs.4,600. What is the share of C?

Or

(b) A, B, C enter in to a partnership and their capitals are in the proportion of $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$. A withdraws half his capital at the end of 4 months. Out of a total annual profit of Rs.847, what will be A's share?

14. (a) A reduction of 20% in the price of sugar enables a purchaser to obtain 4 kg more for Rs.160. What is the reduced price per kg? Also, find its original rate?

Or

(b) A man sells an article at a profit of 20%. If he had bought it at 20% less and sold it for Rs.5 less, he would have gained 25%. Find the cost price of the article.

15. (a) A fraction becomes 4 when 1 is added to both numerator and denominator, and it becomes 7 when 1 is subtracted from both the numerator and denominator. What is the numerator of the fraction?

Or

(b) The sum of squares of two numbers is 80 and the square of their difference is 36. Find the product of the numbers.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $\frac{x}{2y} = \frac{3}{2}$ then find the value of $\frac{2x+y}{x-2y}$.

Or

(b) The average of 5 consecutive numbers is n. If the next two numbers are also included the average will be increased 7 by how much?

17. (a) Two numbers are in the ratio 3:5. If each number is increased by 10, the ratio becomes 5:7. Find the numbers.

Or

(b) Two equal glasses are respectively $\frac{1}{3}$ and $\frac{1}{4}$ full of milk. They are then filled up with water and the contents mixed in a tumbler. Find the ratio of milk and water in the tumbler?

18. (a) A reduction of 21% in the price of wheat enables a person to buy 10.5 kg more for Rs.100. What is the reduced price per kg?

Or

(b) 72% of the students of a certain class took Biology and 44% took Mathematics. If each student took Biology or Mathematics and 40 took both, find the total number of students in the class.

19. (a) 'A' bought 25 kg of rice at the rate of Rs.6 per kg and 35 kg of rice at the rate of Rs 7 per kg. He mixed the two and sold the mixture at the rate of Rs.6.75 per kg. What was his profit or loss in the transaction?

Or

(b) Find the single discount equivalent to a series discount of 20%, 10% and 5%.

20. (a) Divide 50 into two parts so that the sum of their reciprocals is $\frac{1}{12}$.

Or

(b) The sum of squares of two numbers is 80 and the square of their difference is 36. Find the product of the numbers?

———————

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Reg. No. : ....

Code No. : 10430 E **Sub. Code : CNMA 41**

U.G. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fourth Semester

Mathematics

Non Major Elective – MATHEMATICS FOR COMPETITIVE EXAMINATION – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Simple Interest on Rs. 5664 at $13\frac{3}{4}$% per annum for 9 months is Rs. ——————

(a) 600 (b) 584.10

(c) 584 (d) 684.10

2. The compound Interest on Rs. 2800 for $1\frac{1}{2}$ years at 10% per annum is ——————

(a) Rs. 441.35 (b) Rs. 436.75

(c) Rs. 434 (d) Rs. 420

3. Dilip can reap a field in 9 days, which Ram alone can reap in 12 days. In how many days, both together, can reap this field?

(a) $5\frac{1}{2}$ (b) $6\frac{1}{2}$

(c) $5\frac{1}{7}$ (d) $5\frac{3}{4}$

4. A and B can do a piece of work in 6 days and A alone can do it in 9 days. The time taken by B alone to do the work is —————— days

(a) 18 (b) 15

(c) 12 (d) $7\frac{1}{2}$

5. Convert 45 km/hr is to metres /sec

(a) 12.5 m/sec (b) 13.5 m/sec

(c) 14.5 m/sec (d) 12 m/sec

6. Harish covers a certain distance by car driving at 70 km/hr and he returns back at the starting point at 55 km/hr. Find his average speed for the whole journey

(a) 61 km/hr (b) 66 km/hr

(c) 61.6 km/hr (d) 66.6 km/hr

7. If the rent for grazing 40 cows for 20 days is Rs. 370 how many cows can graze for 30 days on Rs. 111?

(a) 6 (b) 8

(c) 5 (d) 12

8. If 20 men can build a wall 112 metres long in 6 days, what length of a similar wall can be built by 25 men is 3 days?

(a) 140 m (b) 44.8 m

(c) 105 m (d) 70 m

9. One tap can fill a cistern in 2 hours and another can empty the cistern in 3 hours. How long will they take to fill the cistern if both the taps are opened?

(a) 5 hrs (b) 6 hrs

(c) 7 hrs (d) 8 hrs

10. Pipe A can fill a tank in 20 hours while Pipe B alone can fill it in 30 hours and Pipe C can empty the full tank in 40 hours. If all the pipes are opened together, how much time will be needed to make the tank full?

(a) $17\frac{1}{7}$ hrs (b) $18\frac{1}{7}$ hrs

(c) 20 hrs (d) $16\frac{1}{7}$ hrs

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Rs. 800 amounts to Rs. 920 in 3 years at simple interest. If the interest rate is increased by 3%, if would amount to how much?

Or

(b) A sum amounts to Rs. 2916 in 2 years and to Rs. 3149.28 in 3 years at compound interest. What is the sum?

12. (a) A and B can complete a work in 10 days and 15 days respectively. A starts the work and after 5 days B also joins him. In all, the work would be completed in hour many days?

Or

[P.T.O.]

(b) A is thrice as good a works man as B and is therefore able to finish a piece of work in 60 days less than B. Find the time in which they can do it, working together.

13. (a) Ram travels a certain distance of 3 km/hr and reaches 15 min. late. If he travels at 4 km/hr, he reaches 15 min. earlier. Find the distance he travelled.

Or

(b) Two men start together to walk to a certain destination, one of 3.75 km an hour and another at 3 km an hour. The former arrives half an hour before the letter. Find the distance.

14. (a) A contract is to be completed in 56 days and 104 men were set to work, each working 8 hours a day. After 30 days $\frac{2}{5}$ of the work is completed. How many additional men may be employed, so that the work may be completed in time, each man now working 9 hours a day?

Or

(b) If 17 labourers can dig a ditch 26 metres long in 18 days, working 8 hours a day how many more labourers should be engaged to dig a similar ditch 39 metres long in 6 days, each labourer working 9 hours a day?

15. (a) Two pipes A and B can fill a tank in 36 min and 45 min respectively. A waste pipe C can empty the tank in 30 min. First A and B are opened. After 7 min. C is also opened. In how much time the tank is full?

Or

(b) A tank can be filled by one tap in 20 minutes and another in 25 minutes. Both the taps are kept open for 5 minutes and then the second is turned off. In how many minutes more is the tank completely filled?

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Two equal amounts of money are deposited in two banks, each at 15% per annum, for $3\frac{1}{2}$ years and 5 years. If the difference between their interests is Rs. 144, find the sum?

Or

(b) A certain sum on compound interest amounts to Rs. 2809 in 2 years and Rs. 2977.54 in 3 years. Find the sum and rate percent?

17. (a) A can do a piece of work in 10 days while B alone do it in 15 days. They work together for 5 days and the rest of the work is done by C in 2 days. If they get Rs. 450 for the whole work, how should they divide the money?

Or

(b) A and B can do a piece of work in 45 and 40 days respectively. They began the work together, but A leaves after some days and B finished the remaining work in 23 days. After how many days did A leave?

18. (a) A man walking at 3km/hr crosses a square field diagonally in 2 minutes. Find the area of the field?

Or

(b) The distance between two stations A and B is 220 km. A train leaves A towards B at an average speed of 80 km/hr. After half an hour, another train leaves B towards A at an average speed of 100 km/hr. Find the distance of the point where the two trains meet from A.

19. (a) 2 men and 7 boys together complete a certain work in 16 days while 3 men and 8 boys together complete the same work in 12 days. Find in how many days will 8 men and 8 boys together, complete a work twice as big as the previous one?

Or

(b) A contractor employed 30 men to do a piece of work in 38 days. After 25 days, he employed 5 men more and the work was finished one day earlier. How many days he would have been behind, if he had not employed additional men?

20. (b) If two pipes function simultaneously the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?

Or

(b) Three pipes A, B and C can fill a cistern in 6 hours. After working altogether for 2 hours, C is closed and A and B can fill it in 7 hours. Find the time taken by C alone to fill the cistern.

———————

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Reg. No. : ....

Code No. : 10426 E **Sub. Code : CSMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics

Skill Based Subject — VECTOR CALCULUS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If ϕ is a scalar function of u and $\vec{a}$ is a constant vector then $\frac{d}{du}(\phi\vec{a}) =$ ————.

   (a) $\vec{a}\frac{d\phi}{du}$ (b) $\phi\frac{d\vec{a}}{du}$

   (c) ϕ (d) $\vec{a}$

2. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $|\vec{r}| = r$ then ∇r is

(a) 0

(b) r

(c) $\vec{r}$

(d) $\frac{\vec{r}}{r}$

3. If ϕ is such that $\nabla^2 \phi = 0$ then ϕ satisfies ———— equation.

(a) Gauss

(b) Laplace

(c) Stokes

(d) None

4. $\nabla \cdot (\nabla \times \vec{A}) =$ ————

(a) 1

(b) $\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

(c) $\nabla^2 \vec{A}$

(d) 0

5. If $\vec{r} = x\vec{i} + y\vec{j} + 2\vec{k}$ then $\nabla \times \vec{r} =$ ————

(a) 1

(b) 2

(c) 3

(d) 0

6. If $\vec{f} = 2x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla \circ \vec{f} =$ ————

(a) 0

(b) 2

(c) 1

(d) 4

7. $\vec{f} = x^2\vec{i} - xy\vec{j}$ and C is the line joining the points (0, 0) and (1, 1) then $\int_C \vec{f} \circ d\vec{r} =$ ________

(a) $\frac{7}{10}$ (b) $\frac{10}{7}$

(c) 1 (d) 0

8. The necessary and sufficient condition for $\int_a^b \vec{f} \circ d\vec{r}$ to be independent of the path is

(a) $\vec{f} = \nabla \phi$ (b) $\nabla \circ \vec{f} = 0$

(c) $\vec{f} = \frac{\nabla \phi}{|\nabla \phi|}$ (d) $\nabla \times \vec{f} = 0$

9. $\operatorname{div} \vec{f}$ is ________.

(a) $\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$ (b) $\frac{\partial \vec{f}}{\partial x} + \frac{\partial \vec{f}}{\partial y} + \frac{\partial f}{\partial z}$

(c) $\frac{\partial f_1}{\partial x}\vec{i} + \frac{\partial f_2}{\partial y}\vec{j} + \frac{\partial f_3}{\partial z}\vec{k}$ (d) None of these

10. $\iiint_V \nabla \circ \vec{f} dV =$ ________

(a) $\iint_S \vec{f} \circ \vec{n} dS$ (b) $\int_S \vec{f} \circ \vec{n} dS$

(c) $\int_S \vec{f} \circ d\vec{r}$ (d) $\iint_S \nabla \circ \vec{f} dS$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\vec{A}, \vec{B}, \vec{C}$ are functions of the scalar variable u, then derive an expression for $\frac{d}{du}\left[\vec{A}\,\vec{B}\,\vec{C}\right]$.

Or

(b) If $\vec{A}, \vec{B}, \vec{C}$ are functions of the scalar variable u, derive an expression for $\frac{d}{du}\left(\vec{A} \times C\vec{B} \times \vec{C}\right)$.

12. (a) Prove that $\nabla\left(\frac{\phi}{\psi}\right) = \frac{\psi \nabla \phi - \phi \nabla \psi}{\psi^2}$.

Or

(b) If $\vec{F} = (x + 2y + az)\,\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational find the values of a, b, c.

13. (a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $P(1, -2, -1)$ in the direction of PQ where Q is $(3, -3, -2)$.

Or

(b) If $\vec{u}, \vec{v}$ are vector point functions then prove that $\nabla \cdot (\vec{u} \times \vec{v}) = \vec{v} \cdot curl\,\vec{u} - \vec{u} \cdot curl\,\vec{v}$.

14. (a) Evaluate $\int_C \phi \, ds$ where C is the curve $x = t$, $y = t^2$, $z = (1-t)$ and $\phi = x^2 y(1+z)$ from $t = 0$ to $t = 1$.

Or

(b) If S is the surface of the sphere $x^2 + y^2 + z^2 = 1$, evaluate $\iint_S \left(x\vec{i} + 2y\vec{j} + 3z\vec{k}\right) \cdot dS$.

15. (a) Evaluate $\iiint_V \nabla \cdot \vec{F} dV$, where

$\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and V is the volume enclosed by the cube $0 \le x, y, z \le 1$.

Or

(b) Evaluate $\int_C \left(e^x dx + zy\, dy - dz\right)$ by Stoke's theorem where C is the curve $x^2 + y^2 = 4$, $z = 2$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove the following :

$$\nabla(\vec{u} \cdot \vec{v}) = (\vec{v} \cdot \nabla)\vec{u} + (\vec{u} \cdot \nabla)\vec{v} + \vec{v} \times curl\, \vec{u} + \vec{u} \times curl\, \vec{v}$$

Or

(b) Prove that $\nabla^2 f(r) = \frac{\partial^2 f}{\partial r^2} + \frac{z}{r}\frac{\partial f}{\partial r}$.

17. (a) Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of vector $2\vec{i} - \vec{j} - 2\vec{k}$.

Or

(b) (i) If $\nabla\phi = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ then find ϕ.

(ii) Prove that $\nabla \cdot (\vec{u} \times \vec{v}) = \vec{v} \cdot (\nabla \times \vec{u}) - \vec{u} \cdot (\nabla \times \vec{v})$

18. (a) Evaluate $\int_C \vec{f} \circ d\vec{r}$ where $\vec{f} = xy\vec{i} + (x^2 + y^2)\vec{j}$ and C is the rectangle in the xy-plane bounded by the lines $y = 2$, $x = 4$, $y = 10$, $x = 1$.

Or

(b) Evaluate $\iiint xyz\, dx\, dy\, dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

19. (a) Verify Green's theorem for $\int_C (x - 2y)\, dx + x dy$ where C is the circle $x^2 + y^2 = 1$.

Or

(b) Evaluate $\iint_S \vec{f} \cdot \vec{n} dS$ where

$\vec{f} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

20. (a) Verify Gauss divergence theorem for the vector function $\vec{F} = 2xz\vec{i} + yz\vec{j} + z^2\vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Verify Stoke's theorem when $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$ where S is the surface of the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$.

(8 pages) Reg. No. : ....

Code No. : 10427 E Sub. Code : CSMA 41

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Fourth Semester

Mathematics — Skill Based Subject

TRIGONOMETRY, LAPLACE TRANSFORMS AND FOURIER SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. 1 radian = ——— degree.

(a) 52.79 (b) 57.29

(c) 59.27 (d) 59.29

2. If $\tan\theta = \frac{1}{15}$ then $\theta \simeq$ ———

(a) 3°47' (b) 3°41'

(c) 3°49' (d) 3°94'

3. The value of $\tanh^{-1} x$ is ———

(a) $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$ (b) $\frac{1}{2}\log\left(\frac{1-x}{1+x}\right)$

(c) $\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)$ (d) $\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)$

4. The value of i^i = ———

(a) $e^{\frac{(4n-1)\pi}{2}}$ (b) $e^{\frac{-(4n+1)\pi}{2}}$

(c) $e^{\frac{-(4n-1)\pi}{2}}$ (d) $e^{(4n-1)\pi}$

5. $L[f'(x)]$ = ———

(a) $f(0) - SL[f(x)]$

(b) $SL[f(x)] - f(0)$

(c) $f'(0) - SL[f(x)]$

(d) $S^2L[f(x)] - Sf(0) - f'(0)$

6. $L^{-1}[F(s+a)]$ = ———

(a) $e^{-ax}L^{-1}[f(s)]$ (b) $e^{ax}L^{-1}[f(s)]$

(c) $e^{ax}L[f(s)]$ (d) $\frac{1}{a}f\left(\frac{s}{a}\right)$

7. $L[xy''] =$ ———

(a) $\frac{d}{dS}\left(S^2L(y) - Sy(0) - y'(0)\right)$

(b) $\frac{d}{dS}\left(S^2L(y) - Sy'(0) - y(0)\right)$

(c) $-\frac{d}{dS}\left(S^2L(y) - Sy(0) - y'(0)\right)$

(d) $-\frac{d}{dS}\left(S^2L(y) - Sy'(0) - y(0)\right)$

8. $L(xy') =$ ———

(a) $-\frac{d}{dS}[SL(y) - y(0)]$

(b) $\frac{d}{dS}[SL(y) - y(0)]$

(c) $\frac{d}{dS}[SL(y) - y'(0)]$

(d) $-\frac{d}{dS}[SL(y) - y'(0)]$

9. The Fourier co-efficient a_0 for $f(x) = x^2$ in $(-\pi, \pi)$ is ———

(a) 0 (b) $\frac{2\pi^3}{3}$

(c) $\frac{2\pi^2}{3}$ (d) $\frac{\pi^3}{3}$

10. For any integer n, the value of $con\,n\pi$ is ———

(a) 1 (b) 0

(c) -1 (d) $(-1)^n$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Prove that

$$2^5 \cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 .$$

Or

(b) Evaluate $\lim_{x\to\frac{\pi}{2}}\left[\frac{\sin x + \cos 2x}{\cos^2 x}\right]$.

[P.T.O.]

12. (a) If $\cos(x+iy) = r(\cos\alpha + i\sin\alpha)$. Then prove that $y = \frac{1}{2}\log\left[\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\right]$.

Or

(b) Sum to infinity the series $1 + \frac{c^2\cos 2\theta}{2!} + \frac{c^4\cos 4\theta}{4!} + \cdots + \infty$.

13. (a) Find $L\left[\frac{1-\cos x}{x}\right]$.

Or

(b) Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$.

14. (a) Using Laplace transform, solve $y'+3y = e^{-2x}$ given $y(0) = 4$.

Or

(b) Solve $(D^2+5D+6)y = e^{-x}$ given that $y(0) = 0$ and $y'(0) = 0$, using Laplace transform.

15. (a) If $f(x) = \begin{cases} -\pi/4 & if \quad -\pi < x < 0 \\ \pi/4 & if \quad 0 < x < \pi \end{cases}$, then find the Fourier series of $f(x)$.

Or

(b) Find the Fourier constant b_1, for the function $x\sin x$ in the half range $0 < x < \pi$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that $\frac{\cos 9\theta}{\cos\theta} = 256\cos^8\theta - 576\cos^6\theta + 432\cos^4\theta - 120\cos^2\theta + 9$.

Or

(b) Prove that

$$\cos^5\theta\sin^4\theta = \left(\frac{1}{2}\right)^8 [\cos 9\theta + \cos 7\theta - 4\cos 5\theta - 4\cos 3\theta + 6\cos\theta].$$

17. (a) If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that

(i) $\theta = \frac{1}{2}n\pi + \frac{\pi}{4}$

(ii) $\phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$.

Or

(b) Sum the series to infinity

$$x\sin\theta + \frac{x^3}{3}\sin 3\theta + \frac{x^5}{5}\sin 5\theta + \cdots + \infty.$$

18. (a) Find

(i) $L\left[t^2 + \cos 2t \cos t + \sin^2 2t\right]$

(ii) $L\left[xe^{-x}\cos x\right]$.

Or

(b) Find

(i) $L^{-1}\left[\frac{s}{(s+2)^2}\right]$

(ii) $L^{1}\left[\frac{s^2}{(s-1)^2}\right]$.

19. (a) Solve $y''-4y'+4y = x$ given that $y(0) = 0$ and $y'(0) = 1$ using Laplace transform.

Or

(b) Using Laplace transform, solve the equation $xy''-(2x+1)y'+(x+1)y = 0$ given that $y(0) = 0$.

20. (a) Find the Fourier series of $f(x) = |\sin x|$ in $(-\pi, \pi)$ of periodicity 2π.

Or

(b) Prove that the function $f(x) = x$ can be expressed in a series of cosine in $0 \le x \le \pi$ as $x = \frac{\pi}{2} - \frac{\pi}{4}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right]$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{8}$.

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Reg. No. : ....

Code No. : 10072 E **Sub. Code : SAMA 11/ AAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

First/Third Semester

Mathematics — Allied

ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2017-2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The n^{th} degree equation $f(x)=0$ cannot have more than ——— roots.

   (a) 4 (b) 6

   (c) 7 (d) n

2. The equation $x^5+4x^4+3x^3+3x^2+4x+1=0$ is ——— equation.

   (a) Quadratic (b) Biquadratic

   (c) Reciprocal (d) None of these

3. How many imaginary roots will occur for the equation $x^6+3x^2-5x+1=0$?

   (a) Atmost four (b) Exactly four

   (c) Atleast four (d) None of these

4. Remove the fractional co-efficients from $x^3-\frac{1}{4}x^2+\frac{1}{3}x-1=0$ multiply the roots by

   (a) 4 (b) 3

   (c) –3 (d) 12

5. The characteristic equation of $\begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$ is

   (a) $\lambda^2-2\lambda-1=0$ (b) $\lambda^2+2\lambda-1=0$

   (c) $\lambda^2-2\lambda+1=0$ (d) $\lambda^2+2\lambda+1=0$

6. The eigen values of $\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ are ———

   (a) $-1, 2$ (b) $1, 2$

   (c) $-1, -2$ (d) $1, -2$

7. General solution of $y = xp + ap^{-1}$ is $y =$ ______

(a) $cx + a$

(b) $2xc + a$

(c) $xc + ac^{-1}$

(d) $yx + ac^{-1}$

8. Partial differential equation from $z = ax + by + a^2$

(a) $z = px + py + a^2$ (b) $z = qx + py + a^2$

(c) $z = px + qy + a^2$ (d) None of these

9. $L[x^2] =$ ______

(a) $\frac{1}{S^2}$ (b) $\frac{2}{S^3}$

(c) $\frac{3}{S^4}$ (d) $\frac{4}{S^5}$

10. $L^{-1}\left[\frac{1}{S}\right] =$

(a) 1 (b) x

(c) e^{ax} (d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve $x^4 + 2x^2 - 16x + 77 = 0$ given that one root is $-2 + i\sqrt{7}$.

Or

(b) Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$ given that the roots are in Arithmetic progression.

12. (a) Transform the equation $x^4 + x^3 - 3x^2 + 2x - 4 = 0$ whose roots are each diminished by 2.

Or

(b) Solve the equation $x^3 - 4x^2 - 3x + 18 = 0$ given that two of its roots are equal.

13. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$.

Or

(b) Using Cayley-Hamilton theorem find the inverse of $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

14. (a) Eliminating the arbitrary functions f and g from $z = f(x+ay) + g(x-ay)$ form a partial differential equation.

Or

(b) From the partial differential equation by eliminating the arbitrary constants from $Z = axe^{y} + a^{2}e^{2y} + b$.

15. (a) (i) Prove that $L[\cosh ax] = \frac{s}{s^{2}-a^{2}}$.

(ii) Prove that

$$L[f'(x)] = s^{2}L[f(x)] - sf(0) - f'(0).$$

Or

(b) Find $L[\sin 2t \sin 3t]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that the roots of the equation $x^{3} + px^{2} + qx + r = 0$ are in Arithmetic progression if $2p^{3} - 9pq + 27r = 0$.

Or

(b) Solve $4x^{4} - 20x^{3} - 33x^{2} - 20x + 4 = 0$.

17. (a) Find correct to two places of decimals the root of the equation $x^{4} - 3x + 1$ that lies between 1 and 2 by Newton method.

Or

(b) Find by Horner's method, the positive root of $x^{3} - 3x + 1 = 0$, lies between 1 and 2. Calculate it to three places of decimals.

18. (a) Find the eigen value and eigen vectors of $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

Or

(b) Verify Cayley-Hamilton theorem for $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

19. (a) Solve $y - 2px = f(xp^{2})$.

Or

(b) Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$.

20. (a) Find $L^{-1}\left[\frac{s^2 - s + 2}{s(s-3)(s+2)}\right]$.

Or

(b) Find $L^{-1}\left[\log \frac{s^2 + 1}{s(s+1)}\right]$.

(8 pages)

Reg. No. : ....

Code No. : 10073 E **Sub. Code : SAMA 21/ AAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second/Fourth Semester

Mathematics

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2017-2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $\bar{f} = (axy - z^3)\bar{i} + (a-2)x^2\bar{j} + (1-a)xz^2\bar{k}$ is irrotational, Then the values of 'a' is ____________.

(a) -4 (b) 4

(c) 2 (d) 0

2. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla.\bar{r} =$ ____________.

(a) 1 (b) 0

(c) 3 (d) $x^2+y^2+z^2$

3. The values of $\iint_R dx\,dy$ where D is the circular disc $x^2+y^2 \le 5$ is ____________.

(a) 5π (b) $2\sqrt{5}\pi$

(c) $\sqrt{5}\pi$ (d) 25π

4. The values of $\iiint_D dx\,dy\,dz$ where D is the region banded by the sphere $x^2+y^2+z^2=9$ is ____________.

(a) 36π (b) $\frac{4\pi}{3}$

(c) 324π (d) $\frac{3\pi}{4}$

5. If C is the straight line joining (0, 0, 0) and (1, 1, 1), then $\int_c \bar{r}.d\bar{r}$ ____________.

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) $\frac{3}{2}$

6. The value of $\int_0^1 \int_0^1 (x^2 + y^2)\, dx\, dy$ is ______________.

(a) $\frac{3}{2}$ (b) 12

(c) $\frac{1}{2}$ (d) 2

7. If S is any closed surface enclosing as volume V and $\bar{f} = ax\bar{i} + by\bar{j} + cz\bar{k}$ then

$\iint_s \bar{f}.\bar{n}\, ds =$ ______________.

(a) 3V (b) $(a+b+c)V$

(c) $(a+b+c)^3 V^3$ (d) 0

8. If R is any closed region of the xy-plane bounded by a simple closed curve C, then $\int_c y dx + x dy$ is ______________.

(a) 1 (b) π

(c) 0 (d) 2π

9. If $f(x)$ is an odd function, ______________.

(a) $f(x) = f(2x)$ (b) $f(x) = f(-x)$

(c) $f(x) = f(-2x)$ (d) $f(x) = -f(-x)$

10. $\int_0^\pi \sin mx \sin x\, dx$ ______________ (if $m = n$)

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{3}$ (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\bar{r}$ is the position vector of any point $P(x, y, z)$, prove that grad $r^n = nr^{n-2}\bar{r}$.

Or

(b) Prove that $div(r^n \bar{r}) = (n+3)r^n$. Deduce that $r^n \bar{r}$ is solenoidal iff $n = -3$.

12. (a) Evaluate $\int_0^1 \int_x^1 (x^2 + y^2)\, dy\, dx$.

Or

(b) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x + 2y)\, dx\, dy$.

13. (a) Evaluate $\int_c \bar{f}.d\bar{r}$ where $\bar{f} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$ and the bounded by $y = 0, y = b, x = 0, x = a$.

Or

(b) Evaluate $\iint_s \bar{f}.\bar{n}\, ds$ where $\bar{f} = (x + y^2)\bar{i} - 2x\bar{j} + 2yz\bar{k}$ and S is the surface of the plane $2x + y + 2z = 6$.

14. (a) Use Green's Thm in a plane to evaluate $\int_c (2x - y)dx + (x + y)dy$ where C is the boundary of the circle $x^2 + y^2 = a^2$ in the xy plane.

Or

(b) Using Stoke's theorem, evaluate $\int_c (\sin x - y)dx = \cos x dy$ where C is the boundary of the triangle whose vertices are (0, 0), $(\pi/2, 0), (\pi/2, 1)$.

15. (a) Find the half range sine series of $f(x) = a$ in $(0, l)$.

Or

(b) Find the half range cosine series of $f(x) = (\pi - x)^2$ in $(0, \pi)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that grade $(\bar{f}.\bar{g}) = \bar{f} \times curl\bar{g} + \bar{g} \times curl\bar{f} + (\bar{f}.\nabla)\bar{g} + (\bar{g}.\nabla)\bar{f}$

Or

(b) If $r = a\cos wt + b\sin wt$ where a, b are constant vectors and w is a constant, prove that $\bar{r} \times \frac{dr}{dt} = w(\bar{a} \times \bar{b})$ and $\frac{d^2r}{dt^2} + w^2\bar{r} = 0$.

17. (a) Find the area of the region D bounded by the parabola $y = x^2$ and $x = y^2$.

Or

(b) Evaluate $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xyz\, dx\, dy\, dz$.

18. (a) Evaluate $\iint_S \bar{f} \cdot \bar{n}\, dS$ where $\bar{f} = (x^3 - yz)\bar{i} - 2x^2y\bar{j} + 2\bar{k}$ and S is the surface of the cube bounded by $x = 0$, $y = 0$, $z = 0$, $x = a$, $y = a$ and $z = a$.

Or

(b) Evaluate $\iint_S \bar{f} \cdot \bar{n}\, dS$ where $\bar{f} = 4x\bar{i} - 2y^2\bar{j} + z^2\bar{k}$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

19. (a) Verify Stoke's theorem for $\bar{f} = (x^2 - y^2)\bar{i} + 2xy\bar{j}$ in the rectangular region $x = 0$, $y = 0$, $x = a$, $y = b$.

Or

(b) Verify Green's theorem for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by the lines $x = 0$, $y = 0$ and $x + y = 1$.

20. (a) Find the half range cosine series for the function $f(x) = x^2$ in $0 \le x \le \pi$ and hence find the sum of the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \ldots$.

Or

(b) Prove that the function $f(x) = x$ can be expanded in a series of cosines in $0 < x < \pi$ as $x = \pi/2 - \frac{4}{\pi}\left(\cos x/1^2 + \frac{\cos 3x}{3^2} + \ldots\right)$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$.

14/06/23 FIN

(8 pages) Reg. No. : FIN

Code No. : 10075 E **Sub. Code : SAST 21/ AAST 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second/Fourth Semester

Mathematics — Allied

STATISTICS — II

(For those who joined in July 2017-2020)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. Paasche' s index number is

   (a) $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$ (b) $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$

   (c) $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \times 100$ (d) $\frac{\Sigma p_0 q_0}{\Sigma p_1 q_1} \times 100$

2. Fisher' s index number is

   (a) arithmetic mean of Laspeyre's and Paasche's index

   (b) median of Laspeyre's and Paasche's index

   (c) harmonic mean of Laspeyre's and Paasche's index

   (d) geometric mean of Laspeyre's and Paasche' s index

3. 5% level of significance is

   (a) $\mu \pm \frac{2\sigma}{\sqrt{n}}$ (b) $\mu \pm 2.58\frac{\sigma}{\sqrt{n}}$

   (c) $\mu \pm 1.96\frac{\sigma}{\sqrt{n}}$ (d) $\mu \pm \frac{\sigma}{\sqrt{n}}$

4. The standard error of the statistic p (sample proportion) is

   (a) $\sqrt{\frac{n}{PQ}}$ (b) $\sqrt{\frac{PQ}{n}}$

   (c) $PQ\sqrt{\frac{1}{n}}$ (d) $\frac{1}{n}\sqrt{PQ}$

5. The test statistic for F-distribution is

   (a) $\frac{S_X^2}{S_Y^2}$ (b) $\frac{S_Y^2}{S_X^2}$

   (c) $S_X^2 S_Y^2$ (d) $S_X S_Y$

6. When expected and observed frequency completely coincide chi-square will be

(a) 0 (b) +1

(c) −1 (d) greater than 2

7. The number of degrees of freedom in a 3 × 3 contingency table is

(a) 8 (b) 9

(c) 4 (d) 1

8. Which of the following is a 2 × 2 Latin square?

(a) $\begin{matrix} A & A \\ B & B \end{matrix}$ (b) $\begin{matrix} A & B \\ B & A \end{matrix}$

(c) $\begin{matrix} B & A \\ B & A \end{matrix}$ (d) $\begin{matrix} A & B \\ A & B \end{matrix}$

9. The R-chart is used to show ________ of the quality produced by the given process.

(a) variability (b) range

(c) sample range (d) standard deviation

10. Which of the following is not an advantages of statistical quality control

(a) Reduction in cost (b) Greater efficiency

(c) Easy to apply (d) Minimum waste

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Describe the characteristics of index numbers.

Or

(b) From the following data of wholesale price construct index numbers for 1991 and 1992 taking 1990 as base.

Commodity	Whole sales price in Rs. Per quintal		
	1990	1991	1992
A	700	750	825
B	540	575	600
C	300	325	310
D	250	280	295
E	320	330	335
F	325	350	360

12. (a) Explain the types of hypothesis.

Or

(b) The means of two single large samples of 1000 and 2000 members are 67.5 and 68 resp. Can the samples be regarded as drawn from the same population of standard deviation 2.5?

[P.T.O.]

13. (a) Write the procedure of t-test for a single mean.

Or

(b) For the following data, test whether this difference is significance or not.

	Sample Size	$\Sigma(x-\bar{x})^2$
(i)	8	84.4
(ii)	10	102.6

5% of F for (7, 9) degrees of freedom is 3.29.

14. (a) The following table gives the results of experiments of 3 varieties of crops 4 in block of plots. Test the significance of difference between the yields of the 3 varieties by preparing the table of analysis of variance.

Variety	Plots			
A	8	4	6	7
B	7	5	5	3
C	2	5	4	4

Or

(b) Three varieties of cows are same age group are treated with four different types of fodders. The yields of milk in deciliters are given below. Perform an ANOV and check whether there is any significant difference between the yields of different varieties of cows due to different types of fodders.

Varieties of Cows	Fodders			
	f_1	f_2	f_3	f_4
C_1	61	63	66	68
C_2	62	64	67	69
C_3	63	63	68	69

15. (a) Define P-chart and write its procedure.

Or

(b) Explain : (i) LTPD (ii) AOQ

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Explain about Ideal Index Number.

Or

(b) For the following data calculate (i) Fishers (ii) Bowley's (iii) Marshall Edgeworth's index numbers.

Commodity	1990		1992	
	Price	Quantity	Price	Quantity
A	2	10	3	12
B	5	16	6.5	11
C	3.5	18	4	16
D	7	21	9	25
E	3	11	3.5	20

17. (a) Describe the procedure to test the equality of population proportions.

Or

(b) Two independent samples give the following results :

Size	Mean	Standard deviation
250	120	12
400	124	14

Test whether the difference between mean is significance or not.

18. (a) A survey of 800 families with four children each revealed the following distribution.

No. of boys :	0	1	2	3	4
No. of girls :	4	3	2	1	0
No. of families :	32	178	290	236	64

Is this result consistent with the hypothesis, that male and female births are equally probable?

Or

(b) The following tables give the classification of 8000 graduates according to sex and employed. Test whether the employment is independent of the sex of the graduates.

	Employed	Non Employed	Total
Male	1480	5720	7200
Female	120	680	800
Total	1600	6400	8000

[Given 5% value of χ^2 (ith 1 d.f.) = 3.841]

19. (a) Describe ANOVA table for one way and two way classification of data.

Or

(b) Perform ANOVA for the following Latin square.

A16	B10	C11	D9	E9
E10	C9	A14	B12	D11
B15	D8	E8	C10	A18
D12	E6	B13	A13	C12
C13	A11	D10	E7	B14

20. (a) Describe double sampling plan and its advantages.

Or

(b) Construct $\overline{X}$ chart for the following data :

Sample No.	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	43	49	37	44	45	37	51	46	43	47
R	5	6	5	7	7	4	8	6	4	6

[Given $A_2 = 0.577$]

(6 pages) **Reg. No. :** ....

Code No. : 10083 E **Sub. Code : SEMA 5 B/ AEMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fifth Semester

Mathematics

Major Elective – DISCRETE MATHEMATICS

(For those who joined in July 2017 – 2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Those statements which contain one or more primary statements and some connectives are called ______________.

   (a) molecular

   (b) composite

   (c) compound statements

   (d) all of the above

2. The statement P is called the ______________ in $P \to Q$.

   (a) antecedent (b) consequent

   (c) tautologies (d) none

3. A sum of the variables and their negations is called as ______________.

   (a) elementary sum (b) elementary product

   (c) normal sum (d) none

4. Which of the following is an example of elementary sums of two variables?

   (a) P (b) $\neg P \vee Q$

   (c) $\neg Q \vee P \vee \neg P$ (d) All of the above

5. Any such disturbance is called ______________.

   (a) encoder (b) decoder

   (c) noise (d) code

6. Any one-to-one mapping of a set S onto S is called a ______________ of S.

   (a) Group (b) Permutation

   (c) Combination (d) Subgroup

7. A ____________ algebra is a complemented, distributive lattice.

(a) Boolean (b) Partial

(c) Ordinary (d) None

8. A lattice is called ____________ if each of its nonempty subsets has a least upper bound and a greatest lower bound.

(a) Sublattice (b) Complement

(c) Complete (d) Bounds

9. What are the numbers using for representing any binary number?

(a) 0 – 9 (b) 0 – 1

(c) 0 – 7 (d) None

10. Add : 100111 and 11011?

(a) 1000010 (b) 100100

(c) 111000 (d) 10101010

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Construct the truth table for $(P \vee Q) \vee \rceil P$.

Or

(b) Write the following statement in symbolic form.

"If either Jery takes calculus or ken takes sociology, then Larry will take English".

12. (a) Symbolize the expression "All the world loves a lover".

Or

(b) Show that $(\exists x)M(x)$ follows logically from the premises $(x)\,(H(x) \rightarrow M(x))$ and $(\exists x)\,H(x)$.

13. (a) Show that A subset $S \neq \phi$ of G is a subgroup of $<G,*>$ iff for any pair of elements $a, b \in S$, $a * b^{-1} \in S$.

Or

(b) Let H be a matrix which consists of K rows and n columns. The prove that the set of words $x =< x_1, x_2, \cdots, x_n >$ which belong to the following set $C = \{x \mid (x \cdot H^t = 0) \bmod 2\}$ is a group code under the operation $\oplus$.

14. (a) Prove that every chain is a distributive lattice.

Or

(b) Prove that $(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$.

15\. (a) Convert $(101010101)_2$ to octal.

Or

(b) Multiply : 1010×1001.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

6\. (a) Construct the truth table for $\neg(P \vee (Q \wedge R)) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$.

Or

(b) Does the formula $(P \rightarrow (P \vee Q))$ is tautology or not?

7\. (a) Obtain the principal disjunctive normal form of $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$.

Or

(b) Show that $\stackrel{S}{\Rightarrow}(\neg Q \wedge (P \rightarrow Q)) \rightarrow \neg P$.

(a) Determine all the proper subgroups of the symmetric group $< S_3, \diamond >$ described in the following table.

$\diamond$	P_1	P_2	P_3	P_4	P_5	P_6
P_1	P_1	P_2	P_3	P_4	P_5	P_6
P_2	P_2	P_1	P_5	P_6	P_3	P_4
P_3	P_3	P_6	P_1	P_5	P_4	P_2
P_4	P_4	P_5	P_6	P_1	P_2	P_3
P_5	P_5	P_4	P_2	P_3	P_6	P_1
P_6	P_6	P_3	P_4	P_2	P_1	P_5

Or

(b) Prove that A code can correct all combinations of k or fewer errors iff the minimum distance between any two code words is at least $2k+1$.

19\. (a) Define : Sub lattice and direct product.

Or

(b) When $< B, *, \oplus, 0, 1 >$ becomes a bounded lattices?

20\. (a) Convert the following to hexa-decimal number.

(i) 1111101101_2

(ii) 11110.01011_2.

Or

(b) Add : 1001.011 and 0100.110

Subtract : $1110 - 0011$.

(7 pages) **Reg. No. :** ....

Code No. : 10085 E **Sub. Code : SEMA 5 D/ AEMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Fifth Semester

Mathematics — Major Elective

OPERATIONS RESEARCH — I

(For those who joined in July 2017–2020)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. A powerful technique to solve the linear programming problems involving three or more decision variables is ————

   (a) Graphical method

   (b) Simplex method

   (c) Two-phase method

   (d) None of these

2. The set of feasible solution to an LPP is a ————

(a) concave set (b) convex set

(c) null set (d) none

3. Number of variables in the dual of 4 constraint primal LPP is ————

(a) 1 (b) 2

(c) 3 (d) 4

4. In the standard form of the LPP, the primal - dual pair is said to be ————

(a) symmetric (b) unsymmetric

(c) equal (d) unequal

5. The transportation problem is balanced if ————

(a) $\Sigma a_i \neq \Sigma b_j$ (b) $\Sigma a_i > \Sigma b_j$

(c) $\Sigma a_i = \Sigma b_j$ (d) $\Sigma a_i < \Sigma b_j$

6. The number of allocated cells in the transportation problem must be ————

(a) $m+n$ (b) $m+n+1$

(c) $m+n-1$ (d) $n+1$

7. Assignment problem is balanced if ————

(a) $m \neq n$ (b) $m = n$

(c) $m+1=n$ (d) $m=n+1$

8. Hungarian method was introduced by ————

(a) G.B. Dantzig (b) D. Konig

(c) D. Henry (d) None

9. The number of possible sequence in n jobs and m machines are ————

(a) $(n!)^m$ (b) $(m!)^n$

(c) $(n)^{m!}$ (d) $(m)^{n!}$

10. Which indicates the time required by a job on each machine?

(a) Idle time (b) Processing time

(c) Elapsed time (d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Explain the standard form of a LPP.

Or

(b) Solve using Graphical method

Max $z = 6x_1 + x_2$

S.t $2x_1 + x_2 \geq 3$, $x_2 - x_1 \geq 0$, $x_1, x_2 \geq 0$.

12. (a) Explain the formulation of a dual problem.

Or

(b) Write the dual of the following LPP.

Min $z = 4x_1 + 6x_2 + 18x_3$

S.t. $x_1 + 3x_2 \geq 3$, $x_2 + 2x_3 \geq 5$, $x_1, x_2, x_3 \geq 0$.

13. (a) Explain the matrix - minima method.

Or

(b) Find an initial basic feasible solution to the following transportation problem using North West corner rule.

	D_1	D_2	D_3	a_i
S_1	8	10	12	900
S_2	12	13	12	1000
S_3	14	10	11	1200
b_j	1200	1000	900	

14. (a) State and prove reduction theorem.

Or

(b) Solve the following assignment problem.

	A	B	C
I	8	7	6
II	5	7	8
III	6	8	7

15. (a) Explain the basic terms used in sequencing.

Or

(b) Explain the processing n jobs and 3 machines.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Write the simplex algorithm.

Or

(b) Solve using two-phase method

Maximize $z = 5x_1 + 3x_2$

S.t. $2x_1 + x_2 \leq 1$, $x_1 + 4x_2 \geq 6$, $x_1, x_2 \geq 0$.

17. (a) State and prove basic duality theorem.

Or

(b) Use duality solve the following

Max $z = 2x_1 + x_2$

S.t. $x_1 + 2x_2 \leq 10$, $x_1 + x_2 \leq 6$, $x_1 - x_2 \leq 2$,

$x_1 - 2x_2 \leq 1$, $x_1, x_2 \geq 0$.

18. (a) Explain the transportation algorithm.

Or

(b) Solve the transportation problem.

	A	B	C	D	Supply
X	6	1	9	3	70
Y	11	5	2	8	55
Z	10	12	4	7	90
Demand	85	35	50	45	

19. (a) Solve the following assignment problem.

	1	2	3
I	9	26	15
II	13	27	6
III	35	20	15
IV	18	30	20

Or

(b) Explain the Hungarian method.

20. (a) Find the optimum sequence for the following.

Jab :	1	2	3	4	5	6
Machine A :	30	120	50	20	90	100
Machine B :	80	100	90	60	30	10

Or

(b) Determine the optimal sequence.

Job :	A	B	C	D	E	F	G
M1 :	3	8	7	4	9	8	7
M2 :	4	3	2	5	1	4	3
M3 :	6	7	5	11	5	6	12

(6 pages) **Reg. No. :** ....

Code No. : 10089 E **Sub. Code : SEMA 6 B**

B.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2023

Sixth Semester

Mathematics – Major Elective

FUZZY MATHEMATICS

(For those who joined in July 2017-2019)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following symbol is used for universal set?

   (a) α (b) a

   (c) A (d) X

2. A set whose members can be labelled by the positive integers is called a ————.

   (a) open set (b) closed set

   (c) countable set (d) uncountable set

3. Let $f : X \to Y$ be an arbitrary crisp function and $A \in \mathcal{F}(X)$. Then ————.

   (a) $A \subset f^{-1}(f(A))$ (b) $A \supset f^{-1}(f(A))$

   (c) $A \subseteq f^{-1}(f(A))$ (d) $A \supseteq f^{-1}(f(A))$

4. Let $A, B \in \mathcal{F}(X)$ and $\alpha, \beta \in [0,1]$. Then $\alpha \le \beta \Rightarrow$

   (a) $\alpha_A \supseteq \beta_A$ (b) $\alpha_A \supseteq \beta +_A$

   (c) $\alpha +_A \supseteq \beta_A$ (d) $\alpha_A \subseteq \beta_A$

5. The standard fuzzy intersection is the only ———— τ-norm.

   (a) Archimedean

   (b) Strictly Archimedean

   (c) Idempotent

   (d) Involutive

6. $u(a,b) = \min(1, a+b)$ is known as ————.

   (a) Standard union (b) Algebraic sum

   (c) Bounded sum (d) Drastic union

7. If $A = [0, 1]$, $B = [1, 2]$ and $C = [-2, -1]$, then $A,(B+C) =$ ————.

   (a) [–1, 1] (b) [–2, 2]

   (c) [1, 1] (d) [2, 2]

8. To Qualify as a fuzzy number, a fuzzy set A on $\mathbb{R}$ must be ————.

(a) convex (b) not convex

(c) sub normal (d) normal

9. The set of vectors X that satisfy all given constraints is called a ————.

(a) Cost vector

(b) Feasible set

(c) Constraint matrix

(d) Right hand - side vector

10. Fuzzy decision making was introduced by

(a) Bellman (b) Blin

(c) Whinston (d) Datiz

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $S(A, B) = \frac{|A \cap B|}{|A|}$.

Or

(b) Prove that the absorption law.

12. (a) Let $A, B \in \mathcal{F}(X)$. Then prove that $\alpha \in [0, 1]$, $A = B$ if and only if $\alpha_A = \alpha_B$ and $A = B$ iff $\alpha +_A = \alpha +_B$.

Or

(b) Let $f : X \to Y$ be an arbitrary crisp function. Then prove that for any $A \in \mathcal{F}(X)$, f fuzzfied by the extension principle satisfies the equation $f(A) = U_{\alpha \in [0,1]} f(\alpha +_A)$.

13. (a) If C is a continuous fuzzy complement, then prove that C has a unique equilibrium.

Or

(b) For all $a, b \in [0, 1]$, prove that max $(a, b) \leq u(a, b) \leq u_{\max}(a, b)$.

14. (a) Let Min and Max be binary operations on R defined by

$$Min(A, B)(z) = \sup_{z=\min(x,y)}[A(x), B(y)]$$

$$Max(A, B)(z) = \sup_{z=\max(x,y)}[A(x), B(y)]$$

respectively. Then prove that, for any $A, B, C \in R$,

$$Min[A, Max(B,C)] = Max[Min(A,B), Min(A,C)$$

Or

(b) Explain the arithmetic operations on intervals.

[P.T.O.]

(a) Define fuzzy linear programming problem.

Or

(b) Solve the following fuzzy linear programming problem:

Max. $Z = 5x_1 + 4x_2$

Such that $\langle 4,2,1\rangle x_1 + \langle 5,3,1\rangle x_2 \leq \langle 24,5,8\rangle$

$\langle 4,1,2\rangle x_1 + \langle 1,5,1\rangle x_2 \leq \langle 12,6,3\rangle$

$x_1, x_2 \geq 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

(a) Prove that a fuzzy set A of $\mathbb{R}$ is convex if and only if $A(\lambda x_1 + (1-\lambda) x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0, 1]$, where min denotes the minimum operator.

Or

(b) Define the following :

(i) α-cut

(ii) Strongly α-cut

(iii) Level set

(iv) Support of a fuzzy set A

(v) The height of A

(vi) Normal and subnormal fuzzy sets.

17. (a) State and prove first decomposition theorem.

Or

(b) For any $A \in \mathcal{F}(X)$, $A = U_{\alpha \Lambda(A)} \alpha_A$ where $\Lambda(A)$ is the set of A, α_A defined by $\alpha_A = \alpha \cdot \alpha_{A(x)}$ and U denotes standard fuzzy union $\Lambda(A) = \{\alpha / \alpha = A(x) \text{ for } x \in X\}$.

18. (a) State and prove first characterization theorem of fuzzy complement.

Or

(b) For all $a, b \in [0, 1]$, prove that $i_{\min}(a, b) \leq i(a, b) \leq \min(a, b)$ where $i_{\min}$ denotes the drastic intersection.

19. (a) State and prove characterization theorem for fuzzy number.

Or

(b) Explain the lattice of fuzzy numbers.

20. (a) Explain the individual decision making.

Or

(b) Explain the multiperson decision making.

(8 pages)

Reg. No. : ....

Code No. : 10091 E **Sub. Code : SEMA 6 D**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Sixth Semester

Mathematics — Major Elective

OPERATIONS RESEARCH—II

(For those who joined in July 2017-2019)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Two person Zero-sum game means that the
   (a) Sum of losses to one player is equal to the sum of gains to other
   (b) Sum of losses to one player is not equal to the sum of gains to other
   (c) Both (a) and (b)
   (d) None of the above

2. The size of the payoff matrix of a game can be reduced by using the principle of ________.
   (a) game inversion
   (b) rotation reduction
   (c) dominance
   (d) game transponce

3. What is concerned with the production of replacement costs and determination of the most economic replacement policy?
   (a) Search theory
   (b) Theory of replacement
   (c) Probabilistic Programming
   (d) none of the above

4. The problem of replacement is felt when job performing units fall ________
   (a) suddenly (b) gradually
   (c) both (a) and (b) (d) none of these

5. Probability of queue size being greater than or equal
   (a) $\frac{p}{1-p}$ (b) p^n
   (c) $1-p$ (d) $\frac{p}{1+p}$

6. For model $(M|M|1):(N \mid F1F0), i = 1$ if P_o = ________

(a) $N+1$ (b) N

(c) $\frac{1}{N+1}$ (d) $\frac{1}{N}$

7. If activity (i,j) is on the critical path, then

(a) $ES_i > LS_i$ (b) $ES_i < LS_i$

(c) $ES_i = LS_i$ (d) $ES_i = 2LS_i$

8. The activity to maintain the proper logic in the network

(a) narrow (b) dummy

(c) circle (d) rectangle

9. The reorder level in EOQ problem with shortages is

(a) $Q_1^0 - Q^0$ (b) $Q^0 + Q_1^0$

(c) $Q_0^0 - Q_1^0$ (d) $\frac{Q_0 - Q_1^0}{2}$

10. For the fundamental EOQ problem, the minimum total annual inventory cost is __________.

(a) $\sqrt{2DC_0C_1}$ (b) $\sqrt{2DC_1/C_s}$

(c) $\sqrt{2DC_0/C_1}$ (d) None of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b)

11. (a) The following matrix represents the payoff to P_1 in a rectangular game between two persons P_1 and P_2. Solve the game.

$$P_1 \overset{P_2}{\begin{pmatrix} 8 & 15 & -4 & -2 \\ 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{pmatrix}}$$

Or

(b) Solve by graphical method

$$\text{Player B} \overset{\text{Player A}}{\begin{pmatrix} 2 & 2 & 3 & -2 \\ 4 & 3 & 2 & 6 \end{pmatrix}}$$

12. (a) Describe the replacement policy of items that deteriorate with time and give the formulae for the total cost Tcn) and average cost ACn).

Or

(b) A truck owner finds from his past records that the maintenance cost per year, of a truck whose purchase price is Rs. 8,000 are as given below :

Year	1	2	3	4	5	6	7	8
Maintenance/cost Rs.	1,000	1,300	1,700	2,200	2,900	3,800	4,800	6,000
Next Sales Price	4,000	2,000	1,200	600	500	400	400	400

Determine at which time it is possible to replace the truck.

[P.T.O.]

13. (a) If $\lambda = 6$, $\mu = 12$, $N = 3$, find $E(n)$, $E(W)$, and $E(m)$.

Or

(b) In the $(M/M/1)$: $(\infty/FCFS)$ model, derive the formula for finding the average number of customers in the system.

14. (a) A project has the following characteristics.

Activity :	1-2	1-3	2-3	2-4	3-4	4-5
Duration (Days) :	20	25	10	12	6	10

Draw the network for the project and find the critical path.

Or

(b) Write briefly on PERT.

15. (a) The demand for a particular item is 18000 units per year. The holding cost per unit is Rs.1.20 per year and the cost of one procurement is Rs. 400. No shortages are allowed and the replacement rate is instantaneous. Determine.

(i) Optimum order quantity.

(ii) Number of orders per year

(iii) Time between orders and

(iv) Total cost per year when the cost of one unit is Re.1.

Or

(b) What are the types of inventory? Why they are maintained?

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Solve graphically the 6 × 2 game

B

		B_1	B_2
A	A_1	1	–3
	A_2	3	5
	A_3	–1	6
	A_4	4	1
	A_5	2	2
	A_6	–5	0

Or

(b) State and prove the theorem for determining the optimum mixed strategies and value of the game of a 2-person zero sum game without saddle point.

17. (a) A machine costs Rs. 10,000. Operating costs are Rs. 500 per year for the first five years. In the sixth and succeeding years operating cost increases by Rs. 100 per year. Assuming a 10% discount rate of money per year, find the optimum length of time to hold the machine before we replace it?

Or

(b) A manufacturer is offered two machines A and B. A is priced at Rs. 5,000 and running costs are estimated at Rs. 800 for each of the first five years, increasing by Rs. 200 per year in the sixth and subsequent years. Machine B, which has the same capacity as A, costs Rs. 2,500 but will have running costs of Rs. 1,200 per year for six years, increasing by Rs. 200 per year thereafter.

If money is worth 10% per year, which machine should be purchased?

18. (a) Define queueing system and explain its basic characteristics. Also give some important applications of queueing theory.

Or

(b) (i) Explain $(M/M/1)(N/FCFS)$

(ii) If for a period of 2 hours in a day (8:10 am) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for the period

(1) The probability that the yard is empty

(2) Average queue length, on assumption that the line capacity of the yard is limited to 4 trains only.

19. (a) Explain the rules of network construction.

Or

(b) Write the algorithm for PERT.

20. (a) Discuss the inventory model with uniform rate of demand, infinite production and no shortages and obtain EOQ.

Or

(b) (i) A contractor has to supply an article 20000 units per day. He can produce 30000 units per day. The cost of holding one unit in stock is Rs. 3 per year and the set up cost per run is Rs. 50. How frequently and what size the product run be made?

(ii) Find the optimum order quantity for a product for which the price breaker are as follows.

Quantity	Unit cost (Rs.)
$0 \le Q_1 < 500$	1,000
$500 \le Q_2 < 750$	9.25
$700 \le Q_3$	8.75

The monthly demand for the product is 200 unit, the cost of storage is 2% of the unit cost and the cost of ordering is Rs. 350.

(6 pages)

Reg. No. : ....

Code No. : 10065 E **Sub. Code : SMMA 53/ AMMA 53**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Fifth Semester

Mathematics – Core

STATICS

(For those who joined in July 2017–2020 only)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

1. The resultant of two forces, P, Q acting along the same line and in the opposite direction is ——————

(a) $\sqrt{P^2+Q^2}$ (b) PNQ

(c) P^2+Q^2 (d) $P+Q$

2. If the resultant of two forces acting at a point with magnitudes 3,5 is a force with magnitude 7, then the angle between them is ——————

(a) 30° (b) 45°

(c) 60° (d) 90°

3. If the line of action of the force passes through a point, then the moment of that force about that point becomes ——————

(a) twice (b) zero

(c) half (d) infinity

4. The moment of force $\overline{F}$ about a point O is ——————

(a) $\overline{r}.\overline{F}$ (b) $\overline{r}\times\overline{F}$

(c) $\overline{F}\times\overline{r}$ (d) $\overline{r}\overline{F}$

5. If three forces acting on a rigid body are in equilibrium then they must be ——————

(a) coplanar (b) perpendicular

(c) not parallel (d) not coplanar

6. If three faces are in equilibrium, and two of them meet at O, then the thrid force —————— O.

(a) is perpendicular to

(b) passes through

(c) does not pass through

(d) is coplanar to

7. The coefficient of friction is equal to ——————

(a) FR (b) R/F

(c) F/R (d) $\tan^{-1}\lambda$

8. Friction is —————— force

(a) active (b) passive

(c) zero (d) resultant

9. The cartesian equation of a catenary is ——————

(a) $y = c\sinh\left(\frac{x}{c}\right)$

(b) $y = c\cos\left(\frac{x}{c}\right)$

(c) $y = c\sin\left(\frac{x}{c}\right)$

(d) $y = c\cosh\left(\frac{x}{c}\right)$

10. The cartesian equation of the catenary shows symmetry about ——————

(a) x–axis (b) y–axis

(c) both x and y axis (d) none

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove Lami's theorem.

Or

(b) State and prove converse of the triangle of forces.

12. (a) Three like parallel forces acting at the vertices of a triangle, have magnitudes proportional to the opposite sides. Show that their resultant passes through the incentre of the triangle.

Or

(b) Find the conditions of equlibrium of three coplanar parallel forces.

13. (a) State and prove three coplanar forces theorem.

Or

(b) The altitude of a right cone is 'h' and the radius of its base is 'a'. A string is fastened to the vertex and to a point on the circumference of the circular base and is then put over a snooth peg, the cone rests with its axis horizontal. Show that the length of the string is $\sqrt{h^2+4a_2}$.

14. (a) A particle of weight 30 kgs resting on a rough horizontal plane is just on the point of motion when acted on by horizontal forces of 6 kg wt and 8 kg wt at right angles to each other. Find the coefficient of friction between the particle and the plane and the direction in which the friction acts.

Or

(b) Find the relation between the coefficient of friction and the angle of friction.

15. (a) Show that if a long claim is thrown over two smooth pegs and is in equilibrium, the free ends must reach the directrix of the catenary formed by it.

Or

(b) Describe the geometrical properties of a common catenary.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Two beads of weights w and w^1 can slide on a smooth circular wire in a vertical plane. They are connected by a light string which subtends an angle 2β at the centre of the circle when the beads are in equilibrium on the upper half of the wire. Prove that the inclination of the string to the horizontal is given by $\tan\alpha = \frac{w \sim w^1}{w + w^1}\tan\beta$.

Or

(b) Derive an analytic expression for the resultant of two forces acting at a point. Discuss its special cases also.

17. (a) Find the resultant of two like parallel forces acting on a rigid body.

Or

(b) State and prove Varigon's theorem of moments.

18. (a) A beam of weight W hinged at one end is supported at the other end by a string so ghat the beam and the string are in a vertical plane and make the same angle θ with the horizon. Show that the reaction at the hinge is $\frac{W}{4}\sqrt{8 + Co\sec^2\theta}$.

Or

(b) A uniform beam of length l and weight W hangs from a fixed point by two strings of lengths a and b. Prove that the inclination of the rod to the horizon is $\sin^{-1}\left(\frac{a^2 - b^2}{l\sqrt{2(a^2 + b^2 - l)}}\right)$. Find also the tension of the strings.

19. (a) A weight can be supported on a rough inclined plane by a force P acting along the plane or by a force Q acting horizontally. Show that the weight is $\frac{PQ}{\sqrt{Q^2\sec^2\lambda - P^2}}$ where λ is the angle of friction.

Or

(b) Find the conditions for the equilibrium of a body on a rough inclined plane under a force parallel to the plane.

20. (a) A uniform chain of length 2l hangs over two small smooth pegs in the same horizontal line and at a distance 2a apart. Show that if h is the sag in the middle, the length of either part of the chain that hangs vertically is $h + l - 2\sqrt{2hl}$.

Or

(b) Derive the equations of the common catenary.

(8 pages)

Reg. No. : ....

Code No. : 10066 E **Sub. Code : SMMA 54/ AMMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fifth Semester

Mathematics – Core

TRANSFORMS AND THEIR APPLICATIONS

(For those who joined in July 2017 – 2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $F\{f(x)\} = \bar{f}(s)$, then $F\{e^{-iax} f(x)\} =$ __________.

   (a) $\bar{f}(s-a)$ (b) $\bar{f}(x-a)$

   (c) $\bar{f}(x+a)$ (d) $\bar{f}(s+a)$

2. $F_c\{f'(x)\} =$ __________.

   (a) $sF_c\{f(x)\} - f(0)$ (b) $sF_s\{f(x)\} - f(0)$

   (c) $F_c\{f(x)\} - f(0)$ (d) $F_s\{f(x)\} - f(0)$

3. $F\{f''(x)\} =$ __________.

   (a) $(is)^2 \bar{f}(s)$ (b) $(is)\bar{f}(s)$

   (c) $(is)^2 \bar{f}(x)$ (d) $(is)\bar{f}(x)$

4. $F_s\{f(x)\cos ax\} =$ __________.

   (a) $\frac{1}{2}\left[\bar{f}_c(s+a) + \bar{f}_c(s-a)\right]$

   (b) $\frac{1}{2}\left[\bar{f}_s(s+a) + \bar{f}_s(a-s)\right]$

   (c) $\frac{1}{2}\left[\bar{f}_s(s+a) + \bar{f}_s(s-a)\right]$

   (d) $\frac{1}{2}\left[\bar{f}_c(s-a) + \bar{f}_c(s+a)\right]$

5. $F_s\{f(x)\} = \bar{f}_s(n) =$ __________.

   (a) $\int_0^l f(x) \sin(n\pi x)\, dx$

   (b) $\int_0^l f(x) \sin(\frac{n\pi x}{l})\, dx$

   (c) $\int_0^l f(x) \cos(n\pi x)\, dx$

   (d) $\int_0^l f(x) \cos(\frac{n\pi x}{l})\, dx$

6. $F_s\{f'(x)\} =$ ___________.

(a) $\frac{n\pi}{l}\bar{f}_s(n)$ (b) $\frac{-n\pi}{l}\bar{f}_s(n)$

(c) $\frac{n\pi}{l}f_c(n)$ (d) $\frac{-n\pi}{l}f_s(n)$

7. $Z\{a^{n-1}\} =$ ___________, if $n \geq 1$.

(a) $\frac{1}{z-a}$ (b) $\frac{z}{z-a}$

(c) $\frac{1}{z+a}$ (d) $\frac{z}{z+a}$

8. $Z(t^2) =$ ___________.

(a) $\frac{T^2 z(z-1)}{(z+1)^3}$ (b) $\frac{T^2 z(z+1)}{(z+1)^3}$

(c) $\frac{T^2 z(z+1)}{(z-1)^3}$ (d) $\frac{T^2 z(z-1)}{(z-1)^3}$

9. $z^{-1}\left\{\frac{z}{z-a}\right\} =$ ___________.

(a) a^{n-1} (b) a^2

(c) $e^{1/z}$ (d) a^n

10. $z^{-1}\left\{e^{a/z}\right\} =$ ___________.

(a) a^{n-1} (b) $\frac{a^{n-1}}{(n-1)!}$

(c) $\frac{a^n}{n!}$ (d) $n!\,a^{n-1}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) $e^{-ax} - e^{-bx} = \frac{2(b^2-a^2)}{\pi}\int_0^\infty \frac{u\sin xu}{(u^2+a^2)(u^2+b^2)}du$ $(a,b>0)$.

Using Fourier integral formula, prove that $e^{-ax} - e^{-bx} = \frac{2(b^2-a^2)}{\pi}\int_0^\infty \frac{u\sin xu}{(u^2+a^2)(u^2+b^2)}du$ $(a,b>0)$.

Or

(b) Find the Fourier transform of $f(x)$, defined as $f(x) = \begin{cases} 1, & for\ |x| < a \\ 0, & for\ |x| > a \end{cases}$. Hence find $f\left\{f(x)\left[1+\cos\frac{\pi x}{a}\right]\right\}$.

12. (a) Find the Fourier sine transform of $f(x)$ defined as $f(x)=\begin{cases}\sin x, & when\, 0<x<a \\ 0, & when\; x>a\end{cases}$.

Or

(b) Use transform methods to evaluate $\int_0^{\infty}\frac{x^2}{(x^2+a^2)(x^2+b^2)}dx$.

13. (a) Find the finite Fourier sine and cosine transforms of $\left(\frac{x}{\pi}\right)$ in $(0,\pi)$.

Or

(b) Find the finite Fourier sine transform of $\cos ax$ in $(0,\pi)$.

14. (a) Find the z-transform of $f(n)=\frac{2n+3}{(n+1)(n+2)}$.

Or

(b) Find the z-transform of $\cos^3 t$.

15. (a) Find $z^{-1}\left\{\frac{1+2z^{-1}}{1-z^{-1}}\right\}$ by the long division method.

Or

(b) Find $z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$, by using Residue theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the equation $(D^2-4D+4)y=xe^{-x}$, $x>0$, given that $y(0)=0$ and $y'(0)=0$.

Or

(b) Solve the one-dimensional heat flow equation $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$ for a rod with insulated sides extending from $-\infty$ to ∞ and with initial temperature distribution given by $u(x,0)=f(x)$.

17. (a) Find the Fourier sine and cosine transforms of x^{n-1}. Hence deduce that $\frac{1}{\sqrt{x}}$ is self-reciprocal under both the transforms. Also find $F\left\{\frac{1}{\sqrt{|x|}}\right\}$.

Or

(b) Find $F_c\left(e^{-a^2x^2}\right)$ and hence find $F_s\left(xe^{-a^2x^2}\right)$.

18. (a) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, 0<x<10$, using finite Fourier transforms, given that $u(0,t)=0$, $u(10, 4)=0$, for $t>0$ and $u(x,0)=10x-x^2$ for $0<x<10$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}, 0<x<l$, using finite Fourier transforms, given that $\frac{\partial u}{\partial x}(0,t)=0$, $\frac{\partial u}{\partial x}(l,t)=0$, for $t>0$ and $u(x,0)=kx$, for $0<x<l$.

19. (a) Find the z-transform of the following functions.

(i) $\cos wt$,

(ii) $\sin wt$,

(iii) $e^{-at}\cos bt$,

(iv) $e^{-at}\sin bt$.

Or

(b) Find the z-transform of $f(n)*g(n)$, where

(i) $f(n)=\left(\frac{1}{2}\right)^n$ and $g(n)=\cos n\pi$

(ii) $f(n)=\begin{cases}\left(\frac{1}{3}\right)^n, & for\ n\geq 0\\ \left(\frac{1}{2}\right)^{-n}, & for\ n<0\end{cases}$ and

$g(n)=\left(\frac{1}{2}\right)^n U(n)$.

20. (a) Find $z^{-1}\left\{\frac{z^2}{(z+2)(z^2+4)}\right\}$, by the method of residues.

Or

(b) Solve the equation $y_{n+2}-7y_{n+1}+12y_n=2^n$, given that $y_0=y_1=0$.

(7 pages)

Reg. No. :.......................................

Code No. : 10067 E **Sub. Code : SMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 – 2019)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

1. The real part of the function $f(z) = z\,\bar{z}$ is ————.

   (a) x^2 (b) $x + y$

   (c) $x^2 + y^2$ (d) xy

2. If $f(z) = u + iv$ is analytic and $f(z) \neq 0$, then $\nabla^2 \log|f(z)| =$ ————.

   (a) $|f'(x)|^2$ (b) $\log|f'(x)|$

   (c) 1 (d) 0

3. The length of the circle $z = a + re^{it}$, $0 \le t \le 2\pi$ is ————

   (a) $2\pi r$ (b) πr^2

   (c) 2π (d) $2r$

4. The value of $\int_C \frac{z\,dz}{z^2 - 9}$, $C : |z| = 2$ is ————.

   (a) $4\pi i$ (b) πi

   (c) 0 (d) $2\pi i$

5. The singular point of $f(z) = \tan z$ is ————.

   (a) $\frac{\pi}{2} + n\pi,\ n \in z$ (b) $\frac{\pi}{2}$

   (c) $\frac{\pi}{2} + n\pi,\ n \in \mathbb{N}$ (d) 0

6. Residue of the function $f(z) = \frac{e^z}{z^2}$ at $z = 0$ is ————.

   (a) $1/2$ (b) 1

   (c) 0 (d) $\frac{1}{z}$

7. Where we evaluated the $\int_0^{2\pi} f(\cos\theta, \sin\theta)\,d\theta$, type of integrals?

   (a) $|z| > r$ (b) $|z| < r$

   (c) $|z| = r$ (d) $|z| = 1$

8. In the integral type $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + a^2)^2}\,dx$, the value of $f(z)$ is ————.

   (a) $\frac{1}{(z^2 + a^2)^2}$ (b) $\frac{e^{iz}}{(z^2 + a^2)^2}$

   (c) $\frac{\cos z}{(z^2 + a^2)^2}$ (d) $\frac{\cos z}{(z^2 + a^2)^3}$

9. The fixed point of the transformation $w = z + a$ is

   (a) None (b) 1

   (c) 0 (d) ∞

10. A bilinear transformation having ∞ as the only fixed point is ————.

    (a) inversion

    (b) translation

    (c) contraction

    (d) magnification

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove complex form of $C - R$ equations.

    Or

    (b) Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are both hormonic, but $u + iv$ is not analytic.

12. (a) Evaluate $\int_C |z|^2\,dz$, where C is the square with vertices $(0,0), (1,0), (1,1)$ and $(0,1)$.

    Or

    (b) State and prove Morera's theorem.

13. (a) Expand $f(z) = \sin z$ in a Taylor series about $z = \pi/4$.

    Or

    (b) Evaluate $\int_C \frac{z^2\,dz}{(z-2)(z+3)}$, where C is the circle $|z| = 4$.

[P.T.O.]

14\. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$.

Or

(b) Evaluate $\int_0^{\infty} \frac{dx}{x^2 + 1}$.

15\. (a) Find the image of the circle $|z - 3i| = 3$ under the map $w = \frac{1}{2}$.

Or

(b) Show that the transformation $w = \frac{5 - 4z}{4z - 2}$ maps the unit circle $|z| = 1$ into a circle of radius unity and centre $-\frac{1}{2}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16\. (a) Show that $u = \log\sqrt{x^2 + y^2}$ is harmonic and determine its conjugate and hence find the corresponding analytic function $f(z)$.

Or

(b) If $f(z)$ is analytic, then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

17\. (a) State and prove Cauchy's integral formula.

Or

(b) Evaluate $\int_C \frac{e^z}{(z+2)(z+1)^2} dz$, where C is $|z| = 3$.

18\. (a) Expand $\frac{1}{(z-1)(2-1)}$ as a power series in z in the regions

(i) $|z| < 1$

(ii) $1 < |z| < 2$

(iii) $|z| > 2$

Or

(b) State and prove Rouche's theorem.

19\. (a) Prove that $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1 - a^2}}, (-1 < a < 1)$.

Or

(b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$.

20\. (a) Find the bilinear transformation which maps the points $-1, 1, \infty$ respectively on $-i, -1, i$.

Or

(b) Find the image of the strip $2 < x < 3$ under $w = \frac{1}{z}$.

Reg. No. : ....

Code No. : 10068 E **Sub. Code : SMMA 62**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics – Core

NUMBER THEORY

(For those who joined in July 2017-2019 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The value of $\binom{n}{n}$ is ________.

   (a) 0 (b) n

   (c) 1 (d) $n!$

2. The value of $1!$ is ________.

   (a) 1 (b) 0

   (c) n (d) none

3. gcd $(-5, 5)$ is ________.

   (a) -5 (b) 5

   (c) 1 (d) 25

4. For any positive integers a and b the value of gcd (a,b) lcm (a,b) is ________.

(a) ab (b) $a+b$

(c) a (d) b

5. Repunit R_n consisting of n consecutive ________.

(a) 2's (b) 1's

(c) 3's (d) 0's

6. The value of $\pi_{4,1}(89)$ is ________.

(a) 13 (b) 10

(c) 8 (d) 9

7. Which one of the following is wrong?

(a) $-12 \equiv 2 \pmod 7$ (b) $13 \equiv 6 \pmod 7$

(c) $91 \equiv 0 \pmod 7$ (d) $82 \equiv 4 \pmod 7$

8. If $4x \equiv 0 \pmod{12}$, the value of x is ________.

(a) 2 (b) 3

(c) 4 (d) 5

9. Which one is the smallest pseudo prime to the base 5?

(a) 91 (b) 217

(c) 247 (d) 341

10. If p is a prime, then $(p-1)! \equiv -1$ ________.

(a) mod 1 (b) mod p

(c) mod $p-1$ (d) none

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the first principle of finite induction.

Or

(b) State and prove Pascal's rule.

12. (a) State and prove Euclid's lemma.

Or

(b) Find the gcd (12378, 3054).

13. (a) If p is a prime and $p \mid ab$, then prove that $p \mid a$ or $p \mid b$.

Or

(b) Prove that there is an infinite number of primes.

14. (a) Solve $18x \equiv 30 \pmod{42}$.

Or

(b) If $ca \equiv cb \pmod n$ and $d = \gcd(c,n)$, prove that $a \equiv b \pmod{n/d}$.

15. (a) State and prove Wilson theorem.

Or

(b) Factor the number 12499 using Fermat's method.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Archimedean property.

Or

(b) State and prove binomial theorem.

17. (a) State and prove Division Algorithm.

Or

(b) Solve the linear diophantine equation $172x + 20y = 1000$.

18. (a) Prove that the number $\sqrt{2}$ is irrational.

Or

(b) If p_n is the n[th] prime number, then prove that $p_n \leq 2^{2^{n-1}}$.

19. (a) State and prove Chinese remainder theorem.

Or

(b) Solve $7x + 3y \equiv 10 \pmod{16}$

$$2x + 5y \equiv 9 \pmod{16}.$$

20. (a) State and prove Fermat's theorem.

Or

(b) Clarify the proof of Wilson theorem with $p = 13$.

———————

(6 pages) **Reg. No. :** ....

Code No. : 10069 E **Sub. Code : SMMA 63**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics – Core

GRAPH THEORY

(For those who joined in July 2017-2019)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In a (p, q) complete graph $q =$ ————.

(a) $\frac{p(p-1)}{2}$

(b) $p(p-1)$

(c) $\frac{p-1}{2}$

(d) $p-1$

2. If G_1 is a (p_1, q_1) graph and G_2 is a (p_2, q_2) graph, then $G_1 \cup G_2$ is a ———— graph.

(a) $(p_1 p_2, q_1 q_2)$

(b) $(p_1 + p_2, q_1 q_2)$

(c) $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$

(d) $(p_1 + p_2, q_1 + q_2)$

3. Which one of the following is a graphic sequence?

(a) (7, 6, 5, 4, 3, 2, 1, 1)

(b) (7, 6, 5, 4, 3, 2, 1)

(c) (7, 6, 5, 4, 3, 2, 2, 2, 1)

(d) (7, 6, 5, 4, 3, 1, 0, 0)

4. In any graph ————.

(a) $\lambda \le k \le \delta$ (b) $\delta \le \lambda \le k$

(c) $k \le \lambda \le \delta$ (d) $k \le \delta \le \lambda$

5. Which one of the following is an Eulerian graph?

(a) k_2 (b) k_3

(c) k_4 (d) k_6

6. If G is a (p, q) tree, $q =$ ————.

(a) $p-1$ (b) $p+1$

(c) p (d) $\frac{p(p-1)}{2}$

7. The largest complete plane graph is ————.

(a) k_3 (b) k_4

(c) k_5 (d) k_6

8. Chromatic number of $\overline{K}_p =$ ————.

(a) 1 (b) p

(c) $p-1$ (d) 0

9. $f(k_2, \lambda)$

(a) λ (b) λ^2

(c) $\lambda(\lambda-1)$ (d) $\lambda^2(\lambda-1)$

10. In any diagraph, ————.

(a) $\Sigma d^+(v) = p$ (b) $\Sigma d^-(v) = p$

(c) $\Sigma d^+(v) = 2q$ (d) $\Sigma d^-(v) = q$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\delta \le \frac{2q}{p} \le \Delta$.

Or

(b) Prove that in any graph, the number of vertices of odd degree is even.

12. (a) Prove that a graph with p points and $\delta \ge \frac{p-1}{2}$ is connected.

Or

(b) Prove that a connected graph with at least two points has at least two points which are not cut points.

13. (a) Write Fluery's algorithm.

Or

(b) Draw all trees with 6 vertices.

14. (a) If G is a plane graph in which every face is an n-cycle, prove that $q = \frac{n(p-2)}{n-2}$.

Or

(b) If G is k-critical, prove that $\delta \ge k-1$.

15. (a) Find the chromatic polynomial of C_4.

Or

(b) Prove that every point of an eulerian weak diagraph has equal indegree and outdegree.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

Or

(b) (i) Prove that any self complementary graph has $4n$ or $4n+1$ vertices

(ii) Prove that $\Gamma(G) = \Gamma(\overline{G})$.

17. (a) Prove that a graph with atleast two points is bipartite iff all its cycles are of even length.

Or

(b) Prove that any closed walk of odd length contains a cycle.

18. (a) Prove that $C(G)$ is well defined.

Or

(b) Prove that every tree has a centre with one point or two adjacent points.

19. (a) State and prove Euler's theorem.

Or

(b) Prove that $\chi'(k_n) = n$ if n is odd and $n \neq 1$.

20. (a) For any (p, q) graph G, prove that $f(G, \lambda)$ is a p degree polynomial with constant term zero.

Or

(b) If every edge of a connected graph G is contained in atleast one cycle, prove that the edges of G can be oriented so that the resulting digraph is strongly connected.

(6 pages) **Reg. No. :** ....

Code No. : 10070 E **Sub. Code : SMMA 64/ AMMA 64**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Sixth Semester

Mathematics — Core

DYNAMICS

(For those who joined in July 2017–2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The time of flight of a projectile is ————

(a) $\frac{u\sin 2\alpha}{g}$ (b) $\frac{u^2\sin^2\alpha}{g}$

(c) $\frac{2u\sin\alpha}{g}$ (d) $\frac{u^2\sin^2\alpha}{2g}$

2. The maximum horizontal range of the projectile is ————

(a) $\frac{u}{g}$ (b) $\frac{u^2}{g}$

(c) $\frac{u\sin\alpha}{g}$ (d) $\frac{u^2\sin\alpha}{g}$

3. If the sphere is perfectly elastic $e=1$, then the loss of Kinetic energy is ————

(a) 0 (b) 1

(c) $u\sin\alpha$ (d) $\frac{1}{2}\cos^2\alpha$

4. The ball is perfectly elastic if ————

(a) $v=0$ (b) $v=u$

(c) $u=0$ (d) $u=\sin\alpha$

5. The period of simple harmonic motion is ————

(a) $\frac{2\pi}{\mu}$ (b) $\frac{2\pi}{\sqrt{\mu}}$

(c) $\frac{\pi}{\sqrt{\mu}}$ (d) $\frac{\pi}{\mu}$

6. The displacement of simple harmonic motion is ———

(a) $x = a\cos\sqrt{\mu}t$ (b) $x = a\cos t$

(c) $x = \cos\sqrt{\mu}t$ (d) none of these

7. The radial component of velocity is ———

(a) $r\theta$ (b) $r\dot{\theta}$

(c) $\dot{r}$ (d) $\dot{r}\dot{\theta}$

8. The transverse component of acceleration is ———

(a) $\frac{1}{r}\frac{d}{dt}(r\dot{\theta})$ (b) $\frac{1}{r}\frac{d}{dt}(r^2\theta)$

(c) $\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$ (d) $\frac{1}{r}\frac{d}{dt}(r\theta)$

9. The differential equation of a central orbit is ———

(a) $u + \frac{du}{d\theta} = \frac{p}{h^2u^2}$ (b) $u^2 + \frac{d^2u}{d\theta^2} = \frac{p}{h^2u^2}$

(c) $u + \frac{d^2u}{d\theta^2} = \frac{p}{h^2u^2}$ (d) $u^2 + \frac{d^2u}{d\theta^2} = \frac{p}{hu}$

10. Pedal equation of the circle is ———

(a) $r = 2ap$ (b) $r^2 = 2ap$

(c) $r^2 = 2a^2p$ (d) $r = 2a^2p$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Find the horizontal range of a projectile.

Or

(b) Derive the range on an inclined plane.

12. (a) Explain the Newton's experimental law.

Or

(b) Explain the direct impact of two smooth spheres.

13. (a) Write a short note on simple harmonic motion.

Or

(b) Explain the change of origin in SHM.

14. (a) Derive the radial and transverse components of velocity.

Or

(b) Explain the equation of motion in polar coordinates.

15. (a) Explain the $(p-r)$ equation of the central orbit.

Or

(b) Derive the $(p-r)$ equation of the parabola.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Show that the path of a projectile is a parabola.

Or

(b) Show that for a given velocity of projection the maximum range down an inclined plane of inclination α bears to the maximum range up the inclined plane the ratio $\frac{1+\sin\alpha}{1-\sin\alpha}$.

17. (a) Find the loss of kinetic energy due to oblique impact of two smooth spheres.

Or

(b) A smooth sphere of mass m impinges obliquely on a smooth sphere of mass M which is at rest. Show that if $m = eM$, the directions of motion after impact are at right angles.

18. (a) Explain the geometrical representation of a simple harmonic motion.

Or

(b) Show that the energy of a system executing SHM is proportionaly to the square of the amplitude and of the frequency.

19. (a) Explain the equiangular spiral.

Or

(b) Explain the velocity and acceleration in polar coordinates.

20. (a) Explain the differential equation of central orbits.

Or

(b) Find the law of force towards the pole under which the curve $r^n = a^n \cos n\theta$.

Reg. No. : ....

Code No. : 10071 E   Sub. Code : SMMA 65/ AMMA 65

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics — Core

NUMERICAL METHODS

(For those who joined in July 2017-2020)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Choose the transcendental equation from the following ————.

   (a) $x^3 - 1 = 0$ (b) $x^2 + x + 1 = 0$

   (c) $x = 1$ (d) $e^x - 1 = 0$

2. Regula-Falsi method is also known as ————.

   (a) method of tangents

   (b) method of false position

   (c) back substitution

   (d) forward substitution

3. If $f(x) = x^2 + x + 1$, then taking $h = 1$, $\Delta f(x) =$ ————.

   (a) $x + 2$ (b) $2(x - 1)$

   (c) $2x$ (d) 2

4. Choose the wrong statement from the following ————.

   (a) $E = 1 + \Delta$ (b) $\nabla = E^{-1}$

   (c) $1 - E^{-1} = \nabla$ (d) $\delta = E^{1/2} - E^{-1/2}$

5. If $f(4) = 1$, $f(6) = 3$, then the interpolating polynomial is ————.

   (a) $3x - 1$ (b) $x - 3$

   (c) $x - 3$ (d) $3x - 2$

6. Newton's backward interpolation formula is used when interpolation is required near the ———— of the table.

   (a) beginning   (b) middle

   (c) end   (d) at the average

7. By evaluating $\int_0^1 \frac{dx}{1+x^2}$ by numerical interpolation, we obtain an approximate value of ————

   (a) $\log_e^2$ (b) π

   (c) $\log_{10}^2$ (d) e

8. Error in Simpson's one third rule is of order ________.

(a) h^2 (b) h^4

(c) h (d) $4h$

9. The order of the difference equation $y_{n+1} - 3y_n = 3^n$ is ________.

(a) 2 (b) 0

(c) 1 (d) 3

10. The order of the difference equation is $y_{n+2} - 2y_{n+1} + y_n = 2^n$ is ________.

(a) $n+1$ (b) 2

(c) n (d) $n+2$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Using Newton-Raphson method, find correct to four decimal places, the root between 0 and 1 of the equation $x^3 - 6x + 4 = 0$.

Or

(b) Solve the equations $2x + y + 4z = 12$, $8x - 3y + 2z = 20$, $4x + 11y - z = 33$ by Gauss-elimination method.

12. (a) Prove that $(\Delta(f_i g_i) = f_i \Delta g_i + g_{i+1} \Delta f_i)$.

Or

(b) Evaluate $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)]$ if the interval of differencing is 2.

13. (a) The following are data from the steam table :

Temperature°C :	140	150	160	170	180
Pressure kgf/cm² :	3.685	4.854	6.302	8.076	10.225

Using Newton's formula, find the pressure of the steam for the temperature of 142°.

Or

(b) Using Gauss's backward formula, find the value of sales for the year 1966 given that

Year :	1931	1941	1951	1961	1971	1981
Sales :	12	15	20	27	39	52

14. (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.25$ from the following values of x and y :

x :	1.00	1.05	1.10	1.15
y :	1.00000	1.02470	1.04881	1.07238
x :	1.20	1.25	1.30	
y :	1.09544	1.11803	1.14017	

Or

(b) Use the trapezoidal rule with $h = \frac{1}{4}$ to evaluate $\int_0^1 f(x)dx$ using the table below :

x :	0.000	0.250	0.500	0.750	1.000
$f(x)$:	0.79788	0.77334	0.70413	0.60227	0.48394

15. (a) Solve the difference equation.
$y_{x+2} - 8y_{x+1} + 15y_x = 0$

Or

(b) Solve $u_{x+2} - 6u_{x+1} + 9u_x = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Determine the root of $xe^x - 3 = 0$ correct to three decimal places, using the method of false position.

Or

(b) Solve by Gauss elimination procedure, the equations.
$3.15x - 1.96y + 3.85z = 12.95$,
$2.13x + 5.12y - 2.89z = -8.61$,
$5.92x + 3.05y + 2.15z = 6.88$.

17. (a) Represent the function $x^4 - 12x^3 + 42x^2 - 30x + 9$ and its successive differences in factorial rotation in which the differencing interval $h = 1$.

Or

(b) Obtain the function whose first difference is $x^3 + 3x^2 + 5x + 2$.

18. (a) Find log 337.5, by Laplace Everett formula, for the following data :

x :	310	320	330
$\log x$:	2.4913617	2.5051500	2.5185139
x :	340	350	360
$\log x$:	2.5314789	2.5440680	2.5563025

Or

(b) Prove Lagrange's interpolation formula in the form

$$f(x) = \sum_{r=0}^{n} \frac{\phi(x) f(x_r)}{(x - x_r)\phi'(x_r)}, \text{ where}$$

$$\phi(x) = \prod_{r=0}^{n} (x - x_r) \text{ and } \phi'(x) = \left[\frac{d}{dx}\phi(x)\right].$$

19. (a) Using the following data, find $f'(5)$.

x :	0	2	3	4	7	9
$f(x)$:	4	26	58	112	466	922

Or

(b) Dividing the range into 10 equal parts, find the approximate value of $\int_0^{\pi} \sin x \, dx$ by

(i) Trapezoidal rule (ii) Simpson's rule

0. (a) Solve the difference equation

$y_{n+3} - 3y_{n+1} + 2y_n = 0$ given $y_1 = 0$, $y_2 = 8$ and $y_3 = -2$.

Or

(b) Solve the equation

$y_{n+2} + y_{n+1} - 56y_n = 2^n(n^2 - 3)$.

(7 pages) Reg. No. : ....

Code No.: 5022 **Sub. Code: PMAM 41**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fourth Semester

Mathematics – Core

FUNCTIONAL ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A complete normed linear space is a ———— space.
   (a) Compact
   (b) Banach
   (c) Continuous
   (d) Hilbert

2. The set of all continuous linear transformations of a normed linear space N into R or C according as N is real or complex denoted by N^* is called ————
   (a) Banach space of N
   (b) complement of N
   (c) conjugate space of N
   (d) Hilbert space of N

3. The conjugate space of N^* is called as ———— conjugate.
   (a) second (b) dual of N
   (c) third (d) first

4. The isometric isomorphism $x \to F_x$ is called the ———— of N into N^{**}
   (a) Banach (b) Natural imbedding
   (c) Surjective (d) Injuctive

5. A ———— space is a complex Banach space the whose norm arises from the inner product.
   (a) Hilbert (b) Banach
   (c) Inner product (d) Banach algebra

6. Two vectors x and y in a Hilbert space H are said to be ———— if $(x, y) = 0$
   (a) orthogonal (b) inverse
   (c) complement (d) inner

7. A ———— set in a Hilbert space H is a non empty subset of H which consists of mutually orthogonal unit vectors.
   (a) Hilbert (b) Empty
   (c) Orthonormal (d) Banach

8. The value of T^{**} = ————.
   (a) T (b) T^*
   (c) T^{-1} (d) T_1

9. An operator is ———— if it commutes with its adjoint.
   (a) adjoint (b) normal
   (c) unitary (d) singular

10. The value of det(I) =
    (a) 0 (b) 1
    (c) –1 (d) 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if N is a normed linear space and x_0 is a nonzero vector in N then there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.

Or

(b) Let M be a closed linear space of a normed linear space N. If the norm of a coset $x + M$ in the quotient space N/M is defined by $\|x + M\| = \inf\{\|x + m\| : m \in M\}$ then prove that N/M is a normed linear space.

12. (a) Prove that if B and B' are Banach spaces and if T is a linear transformation of B into B' then T is continuous if and only if its graph is closed.

Or

(b) If P is a projection on a Banach space B, and if M and N are its range and null space then prove that M and N are closed linear subspaces of B such that $B = M \oplus N$.

13. (a) Prove that if x and y are any two vectors in a Hilbert space H, then $|< x, y >| \le \|x\| \, \|y\|$.

Or

(b) Prove that if M is a closed linear space of a Hilbert Space, then $H = M \oplus M^{\perp}$.

14. (a) Prove that if A_1 and A_2 are self adjoint operators on H, then their product A_1A_2 is self adjoint if and only if $A_1A_2 = A_2A_1$.

Or

(b) Prove that if T is an operator on H, for which $(Tx, x) = 0$ for all x then $T = 0$.

15. (a) Prove that if T is normal, then x is an eigen vector of T with eigen value $\lambda \Leftrightarrow x$ is an eigen vector of T^* with eigen value $\overline{\lambda}$.

Or

(b) If T is normal, then prove that the Mi's are pairwise orthongonal.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Hahn banach theorem.

Or

(b) Let N anad N' be normed spaces and T is a linear transformation of N into N'. Then prove that the following conditions on T are all equivalent.

(i) T is continuous.

(ii) T is continuous at the origin.

(iii) There exists a real number $k \geq 0$ with the property that $\|T(x)\| \leq k\|x\|$ for every x in N.

(iv) If $S = \{x : \|x\| \leq 1\}$ is closed unit sphere in N then its image $T(S)$ is a bounded set in N'.

17. (a) State and prove open mapping theorem.

Or

(b) If N is a normed linear space then prove that the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak * topology.

18. (a) If T is an operator on a normed linear space N, then prove that its conjugate T^* defined by $[T^*(f)](x) = f(T(x))$ is an operator on N^* and the mapping $T \to T^*$ is an isometric isomorphism of $\mathfrak{B}(N)$ into $\mathfrak{B}(N^*)$ which reverses products and preserves the identity transformation.

Or

(b) If M is a proper closed linear subspace of a Hubert space H, then prove that there exists a non zero vector z_0 in H such that $z_0 \perp M$.

19. (a) Prove that if $\{e_i\}$ is an ortho normal set in a Hilbert space H, and if x is an arbitrary vector in H then $x - \sum (x, e_i)e_i \perp e_j$ for each j.

Or

(b) Prove that if H be a Hilbert space and f be an arbitrary functional in H^* then there exists a unique vector y in H such that $f(x) = (x, y)$.

20. (a) Prove that if P is a projection on H with range M and null space N then $M \perp N$ if and only if P is self adjoint and in this case $n = M^{\perp}$.

Or

(b) (i) Prove that $\|N^2\| = \|N\|^2$ if N is normal operator on H.

(ii) Also prove that if T is an operator on H, then T is normal iff its real and imaginary parts commute.

———————

(6 pages) Reg. No. : ....

Code No.: 5374 **Sub. Code: ZMAE 21**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics

CLASSICAL MECHANICS – Elective

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If the force acting on a particle are conservative then T + V is conserved. The name of the theorem is
   (a) Energy conservation theorem for a particle
   (b) Conservation theorem for linear momentum of a particle
   (c) Conservation theorem for the angular momentum of a particle
   (d) Conservation theorem for angular momentum

2. A particle is constrained to move along any curve on a given surface is ———— constraint.
   (a) holonomic (b) non-holonomic
   (c) rheonomous (d) scleronomous

3. U in the equation $L = T - U$ is called ————
   (a) kinetic energy
   (b) momentum
   (c) generalized potential
   (d) torque

4. $Q_j = \sum_i \overline{F}_i \frac{\partial \overline{r_i}}{\partial q_j}$ is called ————
   (a) electromagnetic force
   (b) frictional force
   (c) impulsive force
   (d) generalized force

5. The shortest distance between two points on a given surface is called ———— of the surface.
   (a) radius (b) diameter
   (c) geodesic (d) straight line

6. Let f be a function of n independent variable $\dot{y}_i$ and their derivatives $\dot{y}_i$. The equations $\frac{\partial f}{\partial y_i} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}_i}\right) = 0, i = 1,2,3,\ldots n$ is called ————
   (a) Lagrange's equations
   (b) Hamilton's equations
   (c) Euler's Lagrange's equation
   (d) Newton's equation of motion

7. If $e < 1$ and $E < 0$ then the orbit is ————
   (a) Circle (b) Ellipse
   (c) Parabola (d) Hyperbolic

8. The equation $\omega t = \psi - e\sin\psi$ is known as ————
   (a) Lagrange's equations
   (b) Newton's equations
   (c) Hamilton's equations
   (d) Kepler's equation

9. $\frac{1}{2}r^2\dot{\theta}$ is called ————
   (a) linear velocity (b) angular velocity
   (c) areal velocity (d) relative velocity

10. Which of the following theorem is used in deriving Boyle's law for perfect gas?
    (a) Conservation theorem for linear momentum
    (b) Bertrand's theorem
    (c) Virial theorem
    (d) Cartheodory theorem

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that $\frac{dT}{dt} = F.\upsilon$ and if the mass varies with time $\frac{d(mT)}{dt} = F.p$

    Or

    (b) Prove that $M^2R^2 = M\sum_i m_i r_i^2 - \frac{1}{2}\sum_{i,j} m_i m_j r_{ij}^2$

12. (a) Discuss the motion of a bead sliding on a uniformly rotating wire in a force-free space.

    Or

    (b) Explain Dissipation function.

[P.T.O]

13. (a) Explain the minimum surface of revolution.

Or

(b) Find the shortest distance between two points in a plane.

14. (a) Prove that the central force motion is always motion in a plane.

Or

(b) State and prove Virial theorem.

15. (a) Prove that $\tau = 2\pi\sqrt{m/k}\, a^{3/2}$.

Or

(b) Derive Kepler's equation.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Explain Constraints.

Or

(b) State and prove energy conservation theorem of a particle.

17. (a) Discuss Maxwell equations in connection with Lagrange's equation.

Or

(b) Derive Lagrange's equation of motion for Atwood machine.

18. (a) Discuss the problem of finding out the curve for which any line integral has a stationary value.

Or

(b) Explain Brachistochrone problem.

19. (a) Derive the four integral.

Or

(b) Discuss orbits by inverse square law.

20. (a) Define Laplace-Runge-Lenz vector and discuss their properties.

Or

(b) State and prove Kepler's third law of planetary motion.

———————

(8 pages) **Reg. No. : ....**

Code No. : 5381 **Sub. Code : ZMAE 31**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics

Elective – ALGEBRAIC NUMBER THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The linear Diophantine equation $ax+by=c$ has a solution if and only if ________.

   (a) $\gcd(a,c)\mid b$

   (b) $\gcd(a,b)\mid c$

   (c) $\gcd(c,b)\mid a$

   (d) $c\mid \gcd(a,b)$

2. Which of the following Diophantine equation cannot be solved?

   (a) $6x+51y=22$ (b) $33x+14y=115$

   (c) $14x+35y=93$ (d) $11x+13y=21$

3. Let a and b be integers, not both zero. Then a and b are relatively prime iff there exists integers x and y such that

   (a) $1+ax+by$ (b) $2=ax+by$

   (c) $ab=ax+by$ (d) $a-b=ax+by$

4. The Euclidean algorithm is used for finding the

   (a) 1 cm of two integers

   (b) gcd of two integers

   (c) prime numbers

   (d) composite numbers

5. Two integers a and b, not both of which are zero, are said to be relatively prime if

   (a) $\gcd(a,b)=a$ (b) $a\mid b$

   (c) $\gcd(a,b)=1$ (d) $b\mid a$

6. If p is prime, then P^* is the

   (a) sum of all primes that are less than or equal to p

   (b) product of all primes that are less than or equal to p

   (c) sum of squares of all primes that are less than or equal to p

   (d) product of all primes that are greater than or equal to p

7. The Sieve of Eratosthenes is used for finding

   (a) all primes below a given integer

   (b) all even numbers below a given integer

   (c) all odd numbers below a given integer

   (d) all composite numbers below a given integer

8. If n is an odd pseudo prime, then 2^n-1 is

   (a) pseudo prime (b) prime

   (c) irrational (d) not pseudo prime

9. If p is a prime and a is any integer then a^p-a is

   (a) a multiple of p^2 (b) a multiple of $p-1$

   (c) a multiple of $2p$ (d) a multiple of p

10. If m and n are relatively prime integers then $\varphi(mn)=$ ________.

    (a) $\varphi(m)+\varphi(n)$ (b) $\varphi(m)/\varphi(n)$

    (c) $\varphi(m)-\varphi(n)$ (d) $\varphi(m)\varphi(n)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing either (a) or (b).

11. (a) Find all solutions in integers of $2x+3y+4z=5$.

    Or

    (b) Find all solutions in positive integer $15x+7y=111$.

12. (a) Prove that the Diophantine equation $x^4+x^3+x^2+x+1=y^2$ has the integral solutions $(-1,1)$, $(0,1)$, $(3,11)$ and no others.

    Or

    (b) Determine whether the Diophantic equation $x^2-5y^2-91z^2=0$ has a nontrivial integral solution.

13. (a) If we define $r_n = \langle a_0, a_1, \ldots, a_n \rangle$ for all integers $n \ge 0$, then prove that $r_n = h_n/k_n$.

Or

(b) Prove that the two distinct simple continued fractions converge to different values.

14. (a) Let ξ denote any irrational number. If there is a rational number $\frac{a}{b}$ with $b \ge 1$ such that $\left|\xi - \frac{a}{b}\right| < \frac{1}{2b^2}$ then prove that $\frac{a}{b}$ equals one of the convergents of the simple continued fraction expansion of ξ.

Or

(b) Prove that the product of two primitive polynomial is primitive.

15. (a) The norm of a product equal the product of the norms, $N(\alpha\beta) = N(\alpha)N(\beta) \cdot N(\alpha) = 0$ iff $\alpha = 0$. The norm of an integers in $Q(\sqrt{m})$, then prove that $N(\gamma) = \pm 1$ iff γ is a unit.

Or

(b) Prove that the reciprocal of a unit is a unit. The units of an algebraic number field from a multiplicative group.

PART C — (5 × 8 = 40 marks)

Answer ALL questions by choosing either (a) or (b).

16. (a) Let U be an $m \times m$ matrix with integral elements. Then prove that the following are equivalent :

(i) U is unimodular ;

(ii) The inverse matrix U^{-1} exists and has integral elements ;

(iii) U may be expressed as a product of elementary row matrices. $U = R_g R_{g-1} \ldots R_2 R_1$;

(iv) U may be expressed as a product of elementary column matrices, $U = C_1 C_2 \ldots C_{h-1} C_h$.

Or

(b) Find all solutions of the simultaneous congruences $3x + 3z \equiv 1 \pmod 5$, $4x - y + 5z \equiv 3 \pmod 5$.

17. (a) Suppose that $ax^2 + by^2 + cz^2$ factors into linear factors modulo m and also modulo n; that is $ax^2 + by^2 + cz^2 \equiv (\alpha_1 x + \beta_1 y + \gamma_1 z)(\alpha_2 x + \beta_2 y + \gamma_2 z) \pmod m$

$ax^2 + by^2 + cz^2 \equiv (\alpha_3 x + \beta_3 x + \gamma_3 z)(\alpha_4 x + \beta_4 y + \gamma_4 z) \pmod n$.

If $(m, n) = 1$ then prove that $ax^2 + by^2 + cz^2$ factors into linear factors modulo mn.

Or

(b) Determine whether the equation $x^2 + 3y^2 + 5z^2 + 2xy + 4yz + 6zx = 0$ has a nontrivial solution.

18. (a) Prove that the values r_n defined in $r_n = \langle a_0, a_1, \ldots, a_n \rangle$ satisfy the infinite chain of inequalities

$r_0 < r_2 < r_4 < r_6 < \ldots < r_7 < r_5 < r_3 < r_1$.

Or

(b) If $\langle a_0, a_1, \ldots, a_j \rangle = \langle b_0, b_1, \ldots, b_n \rangle$ where these finite continuous fractions are simple, and if $a_j > 1$ and $b_{n>1}$, then prove that $j = n$ and $a_i = b_1$ for $i = 0, 1, \ldots, n$.

19. (a) Prove that the convergents h_n/k_n are successively closer to ξ, that is $\xi - \left|\frac{h_n}{k_n}\right| < \xi - \left|\frac{h_{n-1}}{k_{n-1}}\right|$.

Or

(b) Prove that the continued fraction expansion of the real quadratic irrational number ξ is purely periodic iff $\xi > 1$ and $-1 < \xi' < 0$, where ξ' denotes the conjugate of ξ.

20. (a) Prove that every Euclidean quadratic field has the unique factorization property.

Or

(b) Prove that the fields $Q(\sqrt{m})$ for $m = -1, -2, -3, -7, 2, 3$ are Euclidean and so have the unique factorization property.

———————

(6 pages)

Reg. No. : ....

Code No.: 5364 **Sub. Code: ZMAM 11**

M.Sc (CBCS) DEGREE EXAMINATION, APRIL 2023.

First Semester

Mathematics – Core

ALGEBRA – I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. If G has no nontrivial subgroups, show that G must be __________ of prime order.

(a) Uncountable (b) Finite

(c) Infinite (d) None of these

2. Every subgroup of an abelian group is __________.

(a) right coset (b) last coset

(c) normal (d) not normal

3. Let G be a group and ϕ an automorphism of G. If $a \in G$ is of order $o(a) > 0$, then $o(\phi(a)) =$ __________.

(a) 0 (b) 1

(c) $o(a)$ (d) ∞

4. The number of automorphisms of a cyclic group of order n is __________.

(a) $\varphi(n)$ (b) n

(c) n^2 (d) 1

5. Every permutation is a product of __________ cycles.

(a) 1 (b) 2

(c) 3 (d) 4

6. If $o(G) = p^2$ where p is a prime numbers then G is __________.

(a) normal (b) left coset

(c) right coset (d) abelian

7. The number of p-sylow subgroups in G, for a given prime is of the form __________.

(a) $1 + kp$ (b) $1 - kp$

(c) kp (d) $\frac{1+k}{p}$

8. If $p^m / o(G)$, $p^{m+1} \nmid o(G)$ then G has a subgroup of order __________.

(a) p^2 (b) p^{m-1}

(c) p^m (d) p^{m+1}

9. If $\phi \neq 1 \in G$ where G is an abelian group then $\sum_{g \in G} \phi(g) =$ __________.

(a) 1 (b) 2

(c) ∞ (d) 0

10. If $g_1 \neq g_2$ are in G, G a finite abelian group, then there is a $\phi \in G$ with $\phi(g_1)$ __________ $\phi(g_2)$.

(a) = (b) ≠

(c) > (d) <

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that N is a normal subgroup of G iff $gNg^{-1} = N$ for every $g \in G$.

Or

(b) Suppose G is a group. N a normal subgroup of G; define the mapping ϕ from G to G/N by $\phi(x) = Nx$ for all $x \in G$. Then prove that ϕ is a homomorphism of G onto G/N.

12. (a) Show that J(G) $\approx G/Z$. where J(G) is the group of inner automorphisms of G, and Z is the center of G.

Or

(b) If G is a finite group. and $H \neq G$ is a subgroup of G such that $o(G \nmid i(H)!$ then Prove that H must contain a nontrivial normal subgroup of G. in particular, G cannot be simple.

13. (a) If $o(G) = p^2$ where p is a prime number, then prove that G is abelian.

Or

(b) Show that Every permutation is the product of its cycles.

14. (a) If $p^m / o(G)$, $p^{m+1} X o(G)$ then show that G has a subgroup of order p^m.

Or

(b) Prove that $n(k) = 1 + p + \ldots + p^{k-1}$.

[P.T.O.]

15. (a) If G and G' are isomorphic abelian groups, then prove that for every integers, $G(s)$, and $G'(s)$ are isomorphic.

Or

(b) Suppose that G is the integral direct product of $N_1, \ldots\ldots, N_n$. Then Prove that for $i \neq j$, $N_i \cap N_j = (e)$, and if $a \in N_i, b \in N_j$ then $ab = ba$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that if ϕ is a homomorphism of G into G with kernel K. then K is a normal subgroup of G.

Or

(b) State and prove Cauchy's theorem for Abelian Groups.

17. (a) State and prove Cayley's theorem.

Or

(b) Show that if G is a group, then Prove that $\mathcal{A}(G)$ the set of automorphisms of G. is also a group.

18. (a) Prove that conjugancy is an equivalence relation on G.

Or

(b) Show that the number of conjugate classes in S_n is $p(n)$, the number of partitions of n.

19. (a) State and prove Third part of Sylow's Theorem.

Or

(b) Let G be a finite group and suppose that G is a subgroup of the finite group M. Suppose further that M has a p-sylow subgroup Q. Then Prove that G has a p-sylow subgroup P. In fact. $P = G \cap xQx^{-1}$ for some $x \in M$.

20. (a) Prove that every finite abelian group is the direct product of cyclic groups.

Or

(b) Show that the two abelian groups of order p^n are isomorphic iff they have the same invariants.

(7 pages) Reg. No. : ....

Code No. : 5369 **Sub. Code : ZMAM 21**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Second Semester

Mathematics – Core

ALGEBRA – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If R is a commutative ring and $a \in R$, the $aR = \{ar \mid r \in R\}$ is a ———— ideal of R
   (a) Right ideal (b) Left ideal
   (c) Two-sided ideal (d) None of the above

2. The number of ideals of the ring of rational numbers is ————
   (a) 2 (b) 1
   (c) 0 (d) none of the above

3. The gcd of $3+4i$ and $4+3i$ in $J[i]$ is ————
   (a) $2-i$ (b) 1
   (c) $1+2i$ (d) none of the above

4. The number of units in the ring of complex numbers is ————
   (a) 0 (b) 2
   (c) 1 (d) 4

5. Which of the following is the unique factorization domain?
   (a) $Z[i]$ (b) $Z(\sqrt{-5})$
   (c) (a) and (b) (d) none of the above

6. The content of the polynomial $3x^6+9x-12$ is ————
   (a) 0 (b) 1
   (c) 3 (d) none of the above

7. Let $F[[x]]$ be the ring of formal power series over a field F. Then rad $F[[x]] =$ ————
   (a) 0 (b) 1
   (c) x (d) none of the above

8. Let R be a commutative regular ring. Then the J – radical of a ring R is
   (a) $\{0\}$ (b) $\{1\}$
   (c) R (d) none of the above

9. A ring R is isomorphic to a subdirect sum of ———— if and only if R is without a prime ideal.
   (a) ideals (b) integral domain
   (c) prime ideals (d) none of the above

10. If $R^\wedge \neq \{0\}$ then the annihilator of the set of zero divisors of R is ————
    (a) R (b) $\{0\}$
    (c) $R^\wedge$ (d) none of the above

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\{0\}$ and R are the only two ideals of the commutative ring R with unit element, then prove that R is a field.

    Or

    (b) If U is an ideal of the ring R, then prove that R/U is a ring and is a homomorphic image of R.

12. (a) Let R be a Euclidean ring and $a,b \in R$, If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.

    Or

    (b) Let p be a prime integer and suppose that for some integer c which is relatively prime to p we can find integers x and y such that $x^2+y^2=cp$. Then prove that there exists integers a and b such that $p=a^2+b^2$.

13. (a) State and prove the division algorithm.

    Or

    (b) Define primitive polynomial and prove that product of two primitive polynomials is a primitive polynomial.

14. (a) Let I be an ideal of R. Then prove that $I \subseteq rad\, R$ if and only if each element of the coset $1+I$ has an inverse in R.

    Or

    (b) For any ring R, prove that the quotient ring $R/Rad\, R$ is without prime radical.

15. (a) An element $a \in R$ is quasi-regular if and only if $a \in I_a$, prove .

Or

(b) Prove that if R is a ring R, $R/radR$ is isomorphic to a subdirect sum of fields.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that every integral domain can be imbedded in a field.

Or

(b) Let R and R' be rings and $\phi: R \to R'$ is a homomorphism of R onto R' with kernel U. Then prove that R' is isomorphic to R/U. Also prove that there is a one-to-one correspondence between the set of ideals of R' and the set of ideals of R which contain U and this correspondence can be achieved by associated with an ideal W' in R' the ideal W in R defined by $W = \{x \in R \mid \phi(x) \in W'\}$. With W so defined, R/W is isomorphic to R'/W'. Prove.

17. (a) Define Euclidean ring and prove that $J[i]$ is an Euclidean ring.

Or

(b) The ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R.

18. (a) State and prove the Eisenstein criterion.

Or

(b) If R is a unique factorization domain and if $p(x)$ is a primitive polynomial in $R[x]$, then prove that it can be factored in a unique way as the product of irreducible elements in $R[x]$.

19. (a) Let I be an ideal of the ring R. Further, assume that the subset $S \subseteq R$ is closed under multiplication and disjoint from I. Then prove that there exists an ideal P which is maximal in the set of ideals which contain I and do not meet S; any such ideal is necessarily prime.

Or

(b) If I is an ideal of the ring R, then prove:

(i) $rad(R/I) \supseteq \frac{radR + I}{I}$ and

(ii) Whenever $I \subseteq radR$, $rad(R/I) = (radR)/I$

20. (a) Let $I_1, I_2, ... I_n$ be a finite set of ideals of the ring R. If $I_i + I_j = R$ whenever $i \neq j$, then prove that $R/\bigcap I_i \simeq \Sigma \oplus \left(\frac{R}{I_i}\right)$.

Or

(b) If R is a ring for which $R^u \neq \{0\}$, then

(i) $ann R^u$ is a maximal ideal of R

(ii) $ann R^u$ consists of all zero divisors of R, plus zero

(iii) Whenever R is without prime radical, R forms a field

———————

(8 pages) Reg. No. : ....

Code No. : 5370 **Sub. Code : ZMAM 22**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics – Core

ANALYSIS – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If P^* is a retirement of P than
   (a) $L(P,f,\alpha) \ge L(P^*,f,\alpha)$
   (b) $U(P^*,f,\alpha) \ge U(P,f,\alpha)$
   (c) $L(p^*,f,\alpha) \le U(P^*,f,\alpha)$
   (d) $U(P,f,\alpha) - L(P,f,\alpha) < E$

2. If $a < s < b$, f is bounded on $[a,b]$, f is continuous at s, and $\alpha(x) = I(x-s)$, then $\int_a^b f\,d\alpha$ is
   (a) $f(s)$ (b) $f(0)$
   (c) $\alpha(s)$ (d) $\begin{cases} 0 \text{ if } x \le 0 \\ 1 \text{ if } x > 0 \end{cases}$

3. Form $= 1, 2, \ldots,$ $n = 1,2,3,\ldots,$ let $s_{m,n} = \frac{m}{m+n}$, Then $\lim_{m \to \infty} \lim_{n \to \infty} s_{m,n}$ is
   (a) 1 (b) 0
   (c) ∞ (d) $\frac{m}{m+n}$

4. If $f \in \zeta(x)$, its supremum noun is defined by
   (a) $\|f\| = \sup_{x \in X} |f(x)|$
   (b) $\|f\| = \inf_{x \in X} |f(x)|$
   (c) $\|f\| = \lim_{x \to \infty} |f(x)|$
   (d) $\|f\| = f(x_0)$ where x_0 is any point of x

5. $\{f_n\}$ is uniformly bounded on E is
   (a) there exists a finite valued function ϕ on E such that $|f_n(x)| < \phi(x)$ $(x \in E,\ n = 1,2,\ldots)$
   (b) there exists a number M such that $|f_n(x)| < M$ $(x \in E,\ n = 1,2,\ldots)$
   (c) there exists a sequence of number m_n st $|fn(x)| < m_n\ (x \in E)$ $n = 1,2,3\ldots$
   (d) there exists a function f such that $f_n(x) \to f(x)$ as $n \to \infty$

6. Which one of the following is true
   (a) Every convergent sequence contains a uniformly convergent subsequence
   (b) Every member of an equicontinuous family is uniformly continuous
   (c) The uniform convergence of $\{f_n\}$ impairs the uniform convergence of $\{f_n'\}$
   (d) If $\{f_n\}$ is a uniformly bounded sequence of continuous functions on a compact set E, then there exist a subsequence which converges pointwise on E

7. The uniform closure of set of polynomials on $[a,b]$ is
   (a) the set of polynomials on $[a,b]$
   (b) empty
   (c) the set of functions on $[a,b]$
   (d) the set of continuous functions $[a,b]$

8. Suppose $\sum_1^\infty C_n = A$. Let $f(x) = \sum_0^\infty c_n x^n\ (-1 < x < 1)$ then $\lim_{x \to 1} f(x)$ is
   (a) $c_1 + c_2 + c_n$ (b) 0
   (c) 1 (d) A

9. The Fourier coefficient c_m of f is
   (a) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-imx} dx$ (b) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{imx} dx$
   (c) $\frac{1}{2\pi i}\int_{-\pi}^{\pi} f(x) e^{-imx} dx$ (d) $\frac{1}{2\pi i}\int_{-\pi}^{\pi} f(x) e^{imx} dx$

10. The value of $\Gamma(1/2)$ is
   (a) π (b) $\sqrt{\pi}$
   (c) 1/2 (d) 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\int_{-a}^{b} f d\alpha \le \int_{a}^{-b} f d\alpha$.

Or

(b) State and prove the fundamental theorem.

12. (a) Give an example to show that a convergent series of continuous functions may have a discontinuous sum.

Or

(b) When do we say the sequence $\{f_n\}$ converges uniformly on E to a function f? State and prove a very convenient test for uniform convergence due to weierstress.

13. (a) Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in R(\alpha)$ on $[a,b]$ for $n=1,2,...$ and suppose $f_n \to f$ uniformly on $[a,c]$. Prove that $f \in R(\alpha)$..

Or

(b) If K is a compact metric space, if $f_n \in \zeta(K)$ for $n=1,2,3,...$ and if $\{f_n\}$ converges uniforms on K, prove that $\{f_n\}$ is equicontinuous on K.

14. (a) Define the uniform closure B of an algebra A of bounded functions. Prove that B is a uniformly closed algebra.

Or

(b) Find the limit $\lim_{x\to 0}\frac{b^x-1}{x}(b<0)$.

15. (a) Prove that every nonconstant polynomial with complex coefficients has a complex root.

Or

(b) If $x>0$ and $y>0$, prove that $\int_0^1 f^{x-1}(1-f)^{y-1}dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) When do you say that f is integrable w.r.t α, in the Riemann sense? Prove that $f \in R(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon>0$ there exists a partition P such that $U(p,f,\alpha)-L(p,f,\alpha)<\Sigma$.

Or

(b) Suppose $f \in R(\alpha)$ on $[a,b]$, $m \le f \le M$, ϕ is continuous on $[m,M]$ and $h(x)=\phi(f(x))$ on $[a,b]$. Prove that $h \in R(\alpha)$ on $[a,b]$.

17. (a) If γ' is continuous on $[a,b]$, prove that γ is rectifiable and $\wedge(\gamma)=\int_a^b |r'(t)|dt$.

Or

(b) Suppose $f_n \to f$ uniformly on E, a metric space. Prove that $\lim_{t\to x}\lim_{h\to\alpha} f_n(t)=\lim_{n\to\alpha}\lim_{t\to x} f_n(t)$ where x is a limit point of E.

18. (a) Prove that there exists a real continuous function on the red line which is nowhere differentiable.

Or

(b) If $\{f_n\}$ is a pointwise founded sequence of complex functions on a countable set E, prove that $\{f_n\}$ has a subsequence $\{fn_k\}$ such that $\{fn_k^{(x)}\}$. Converges for every $x \in E$.

19. (a) State and prove that Weierstrass theorem.

Or

(b) State and prove the Afel's theorem for powerserviece.

20. (a) State and prove parseval's theorem.

Or

(b) If f is a positive function on $(0,\propto)$ such that $f(x+1)=xf(x)$, $f(1)=1$ and $\log f$ is convex, then prove that $f(x)=\Gamma(x)$.

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(8 pages) **Reg. No. :** ....

Code No. : 5371 **Sub. Code : ZMAM 23**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics — Core

ADVANCED CALCULUS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The value of $\int_1^2 \sqrt{x}\,dx$ is

(a) $\frac{4\sqrt{2}-2}{5}$ (b) $\frac{4\sqrt{2}-2}{3}$

(c) $\frac{4\sqrt{2}-1}{3}$ (d) $\frac{\sqrt{2}-4}{3}$

2. Let D be the region between the line $y = x$ and the parabola $y = x^2$. Let $f(x, y) = xy^2$. Then $\iint_D f$ is

(a) $\frac{1}{20}$ (b) $\frac{1}{42}$

(c) $\frac{1}{30}$ (d) $\frac{1}{40}$

3. The image of the line $x = 0$ under the transformation $S : \begin{cases} u = x + y \\ v = x - y \\ w = x^2 \end{cases}$ is

(a) $u = v$

(b) the line $u + v = 0$, in the plane $w = 0$

(c) $u = 0$, $v = 0$, $w = 0$

(d) a circle

4. Let T be the linear transformation on R^2 into R^2 specified by the matrix $\begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$. The image of the point $(1, 2)$ is

(a) (0, 3) (b) (3, 0)

(c) (0, –3) (d) (–3, 0)

5. The Jacobian of the transformation $T : \begin{cases} u = x\cos y \\ v = x\sin y \end{cases}$ is

(a) 0 (b) y

(c) x (d) 1

6. If $T : \begin{cases} u = \cos(x + y^2) \\ v = \sin(x + y^2) \end{cases}$ then at (x, y) the Jacobian of T is

(a) 0

(b) $4y\sin(x + y^2)\cos(x + y^2)$

(c) $2y\sin(x + y^2)\cos(x + y^2)$

(d) 2

7. If F is additive on A, the $F(S_1 \cup S_2)$ is

(a) $F(S_1) + F(S_2)$

(b) $F(S_1) + F(S_2) + F(S_1 \cap S_2)$

(c) $F(S_1) + F(S_2) - F(S_1 \cap S_2)$

(d) $F(S_1) + F(S_2) - 2F(S_1 \cap S_2)$

8. The direction of the line through (1, 2, –1) towards (3, 1, 1) is

(a) $\left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right)$ (b) $(2, -1, 2)$

(c) $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ (d) $\left(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}\right)$

9. If $V = Ai + Bj + Ck$ then $curl(V)$ is

(a) $(C_2 - B_3)i + (A_3 - C_1)j + (B_1 - A_2)k$

(b) $(C_2 - B_3)i + (A_2 - C_1)j + (B_2 - A_1)k$

(c) $(C_1 - B_1)i + (C_2 - B_2)j + (A_3 - B_1)k$

(d) $(C_2 - B_3)i - (A_3 - C_1)j + (B_1 - A_2)k$

10. If α is a k form and β any differential form, then $d(\alpha\beta)$ is

(a) $(d\alpha)\beta + (-1)^k \alpha(d\beta)$

(b) $(d\alpha)\beta + (d\beta)\alpha$

(c) $(d\alpha)\beta - \alpha(d\beta)$

(d) $(-1)^k(d\alpha)\beta + (-1)^k \alpha(d\beta)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Let f and g be continuous and bounded on D. Prove that $\iint_D |f|$ exists and $\left|\iint_D f\right| \le \iint_D |f|$.

Or

(b) Show that for $x > 0$

$$\int_0^{\pi/2} \log(\sin^2\theta + x^2\cos^2\theta)\,d\theta = \pi\log\left(\frac{x+1}{2}\right).$$

12. (a) Let $T:\begin{cases} r = xy \\ s = 2x \\ t = -y \end{cases}$, $S:\begin{cases} u = r-s \\ v = st \end{cases}$.

Calculate the products ST and TS. Verify whether $ST = TS$ or not.

Or

(b) Define the differential of a transformation T compute the differential of

$$T:\begin{cases} u = x+6y \\ v = 3xy \\ w = x^2-3y^2 \end{cases} \quad \text{at } (1, 1)$$

13. (a) Discuss the solution of the equations for u and v

$$\begin{cases} x^2 - yu = 0 \\ xy + uv = 0 \end{cases}.$$

Or

(b) Let T be of class C' in an open region D and let E be a closed bounded subset of D. Let dT/p_0 be the differential of T at a point $p_0 \in E$. Prove that

$T(p_0 + \Delta p) = T(p_0) + dT/p_0(\Delta p) + R(\Delta p)$ where $\lim_{\Delta p \to 0} \frac{|R(\Delta p)|}{|\Delta p|} = 0$ uniformly for $p_0 \in E$.

14. (a) If E is a closed bounded subset of Ω at zero volume, prove that $T(E)$ has zero volume.

Or

(b) If γ_1 and γ_2 are smoothly equivalent curves, prove that $L(\gamma_1) = L(\gamma_2)$.

15. (a) If ω is any differential form of class C'', prove that $dd\omega = 0$.

Or

(b) State and prove the divergence theorem for the case of a cube.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If f is continuous on R, prove that $\iint_R f$ exists.

Or

(b) Let R be the rectangle described by $a \le x \le b$, $c \le y \le d$ and let f be continuous on R. Prove that $\iint_R f = \int_a^b dx \int_c^d f(x, y)dy$.

17. (a) Let L be a linear transformation from R^n into R^m represented by the matrix $[a_{ij}]$. Prove that there is a constant B such that $|L(p)| \le B|p|$ for all points p. Also show that the number B is not the smallest number with this property.

Or

(b) Let T be differentiable on an open set D and let S be differentiable on an open set containing $T(D)$. Prove that ST is differentiable on D and if $p \in D$ and $q = T(p)$, then $d(ST)_p = dS/_q\, dT/_p$.

18. (a) Let T be a transformation from R^n into R^n which is of class C' in an open set D, and suppose that $J(p) \neq 0$ for each $p \in D$. Prove that T is locally 1-to-1 in D.

Or

(b) Let T be of class C' on an open set D in n space, taking values in n space. Suppose that $J(p) \neq 0$ for all $p \in D$. Prove that $T(D)$ is an open set.

19. (a) Let F be an additive set function defined on $\mathcal{P}$ and a.c. Suppose also that F is differentiable everywhere, and uniformly differentiable on compact sets, with the derivative a point function f. Prove that f is continuous everywhere and $F(S) = \iint_S f$ holds for every rectangle S.

Or

(b) Define a smooth curve. If γ is a smooth curve whose domain is the interval $[a, b]$. Prove that γ is rectifiable and $L(\gamma)$ is given by the formula $L(\gamma) = \int_a^b |r'(t)|dt$.

20. (a) Prove that $T^*(d\omega) = (d\omega)^* = d(w^*) = dT^*(\omega)$ when (i) ω is a 0-form (ii) ω is any 1-form.

Or

(b) Give a proof of Stoke's theorem by reducing it to an application of Green's Theorem.

(6 pages)

Reg. No. : ....

Code No. : 5372 **Sub. Code : ZMAM 24**

M.Sc.(CBCS) DEGREE EXAMINATION, APRIL 2023

Second Semester

Mathematics

DIFFERENTIAL GEOMETRY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The normal which is perpendicular to the osculating plane at a point is called————

(a) principal normal

(b) binormal

(c) tangent line

(d) curvature

2. The necessary and sufficient condition that a curve be a straight line is that ———— at all points.

(a) t = 0 (b) k = 1

(c) k = 0 (d) c = 0

3. The involute of a circular helix are plane curves, whose planes are ———— to the axis of the cylinder

(a) normal (b) point

(c) centre (d) length

4. If two curves have the same intrinsic equations, then they are ————

(a) equal (b) congruent

(c) parallel (d) constant

5. If the curvature and torsion are both constant, then the curve is————

(a) helix (b) circular helix

(c) sphere (d) constant curve

6. If (l,m) are direction coefficients of a direction on the surface ,then the numbers μ,λ which are ———— to (l,m) are called direction ratio

(a) parallel (b) orthogonal

(c) perpendicular (d) proportional

7. If the parametric curves are ———— then the curve $u = c$ will be geodesic iff $G_1 = 0$.

(a) constant (b) orthogonal

(c) parallel (d) equal

8. If $EE_2 + FE_1 - 2EF_1 = 0$ is satisfied for all values of u and v the parametric curves ———— are all geodesics.

(a) u=constant (b) v=constant

(c) u+v =constant (d) u-v =constant

9. If the orthogonal trajectories of the curve v=constant are geodesics, then______ is independent of u

(a) H/E2 (b) H^2/E

(c) $H/\sqrt{E}$ (d) $\sqrt{H}/E^2$

10. The curvature of a geodesic relative to itself is————

(a) constant (b) parallel

(c) zero (d) equal

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the curvature and torsion are equal for the curve $x = a(3u - u^3), y = 3au^2, z = a(3u + u^3)$

Or

(b) Derive Serret Frenet formula.

12. (a) Find the expression for the curvature and torsion if the curve is drawn on a parabolic cylinder so as to cut all the generators at the same angle.

Or

(b) Find curvature and torsion of an evolute.

13. (a) Find the direction coefficient making an angle $\pi/2$ with the direction coefficient (l,m).

Or

(b) Prove that the metric is invariant.

[P.T.O.]

14. (a) Prove that on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesics is $EE_2 + FE_1 - 2EF_1 = 0$ when v = c for all values of u.

Or

(b) Show that the curves $u + v =$ constant are geodesics on a surface with metric $(1+u^2)du^2 - 2uvdudv + (1+v^2)dv^2$.

15. (a) Prove that the necessary and sufficient condition for a curve on a surface to be a line of curvature is k dr + dN = 0 at each point of the line of curvature.

Or

(b) Discuss the nature geodesic on a plane.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Discuss the curvature and torsion of curve in the three cases.

Or

(b) Calculate the curvature and torsion of the cubic curve given by $r = (u, u^2, u^3)$.

17. (a) State and prove Existence theorem for space curves.

Or

(b) Derive the equation of an involute of a curve.

18. (a) Prove that the curves of family $v^3/u^2 = c$ are geodesics on a surface with metric $v^2du^2 - 2uvdudv + 2u^2dv^2$.

Or

(b) A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoids.

19. (a) Prove that metric is a positive definite quadratic form in du, dv Also discuss the invariance property.

Or

(b) Prove that every helix on a cylinder is a geodesic.

20. (a) State and prove Liovillie's formula.

Or

(b) Prove that the geodesic curvature vector of any curve is orthogonal to the curves.

(7 pages) **Reg. No. :** ....

Code No. : 5373 **Sub. Code : ZMAM 25**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics – Core

RESEARCH METHODOLOGY AND STATISTICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Text books are example of ________ of information.

(a) Primary sources (b) Secondary sources

(c) Constant (d) Variable

2. The length of the abstract may be ________ words.

(a) 100 (b) 200

(c) 300 (d) 400

3. The marginal p.d.f. $f_2(x_2)$ of X_2 in discrete case is ________.

(a) $\sum_{x_1} f(x_1 x_2)$ (b) $\sum_{x_2} f(x_1 x_2)$

(c) $\sum_{x_1} f(x_1 x_1)$ (d) $\sum_{x_1} f(x_2 x_2)$

4. If the joint p.d.f. of the random variable $x_1, x_2, f(x_1, x_2) = x_1 + x_2$, $0 < x_1 < 1$, $0 < x_2 < 10$ elsewhere then the marginal p.d.f. of X_1 is ________.

(a) $x_2 + 1$ (b) $x_2 + \frac{1}{2}$

(c) $x_1 + \frac{1}{2}$ (d) $x_2 + 2$

5. The mean of the gamma distribution is ________.

(a) $\alpha\beta$ (b) $\alpha\beta^2$

(c) $\alpha 2\beta$ (d) $3\alpha\beta$

6. The variance of Chi-square distribution $\chi^2(r)$ is ________.

(a) r (b) $2r$

(c) $3r$ (d) r^2

7. A function of one or more random variables that does not depend upon any unknown parameters is called a

(a) statistic (b) parameter

(c) unit (d) variance

8. If F has an F-distribution with parameters r_1 and r_2 then $1/F$ has an F-distribution with parameters ________ and ________.

(a) $1/r_1, 1/r_2$ (b) r_2, r_1

(c) $1/r_2, 1/r_1$ (d) r_1, r_2

9. The mean of the random sample $x_1, x_2, \ldots x_n$ $n \geq 2$ is ________.

(a) $\frac{\sum x_i}{n}$ (b) $1/n$

(c) x_i/n (d) $X_i * n$

10. If $f(x_1, x_2) = \frac{x_1 x_2}{36}$, $x_1 = 1, 2, 3$, $x_2 = 1, 2, 3$ / 0 elsewhere then $\Pr(x_1 = 2, x_2 = 3)$ is ________.

(a) 0 (b) 1

(c) 1/6 (d) 1/2

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing either (a) or (b).

11. (a) What are the types, methods and techniques used in research methodology?

Or

(b) State the any three research process in flow chart.

12. (a) The joint p.d.f. of the random variable X and Y is $f(x, y) = 6x^2 y$, $0 < x < 1, 0 < y < 1$ and zero elsewhere then find $\Pr(0 < x < 3/4, 1/3 < y < 2)$.

Or

(b) Let the joint p.d.f. of the random variables X_1 and X_2 be $f(x_1, x_2) = x_1 + x_2$, $0 < x_1 < 1, 0 < x_2 < 1$ and zero else where then prove that X_1 and X_2 are dependent.

13. (a) Let X be $\chi^2(10)$. Then find $\Pr(3.25 < X < 20.5)$.

Or

(b) If $(1 - 2t)^{-6}, t < 1/2$ is the m.g.f. of the random variable X then find $\Pr(X < 5.23)$.

14. (a) Let X have the p.d.f. $f(x) = 1/3$, $x = 1, 2, 3$, zero else then find the p.d.f. of $Y = 2X + 1$.

Or

(b) Let X have the p.d.f. $f(x) = 2x, 0 < x < 1$ zero then find the value of the jacobian if $Y = 8x^3$ also the p.d.f. of Y.

15. (a) Let X_1 and X_2 be independent with normal distributions $N(\mu_1, \sigma_2^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Then for the random variable $Y = X_1 - X_2$, find the p.d.f. $g(y)$ of Y.

Or

(b) Let the random variable X_1, X_2 have the same p.d.f. $f(x) = x/6$, $x = 1, 2, 3$, 0 else. Then find $\Pr(X_1 + X_2 = 3)$

PART C — (5 × 8 = 40 marks)

Answer ALL questions by choosing either (a) or (b).

16. (a) Write about bibliography and appendices.

Or

(b) Explain about the review of literature.

17. (a) Let X_1, X_2 and X_3 be three mutually independent random variables and let each have the p.d.f. $f(x) = 2x, 0 < x < 1$, zero elsewhere. The joint p.d.f. of X_1, X_2 and X_3 is $f(x_1)f(x_2)f(x_3) = 8x_1x_2x_3, 0 < x_i < 1, i = 1, 2, 3$, zero elsewhere. Find the expected value of $5X_1X_2X_3 + 3X_2X_3^4$. Also find the p.d.f. of the random variable Y the maximum of X_1, X_2 and X_3.

Or

(b) Let X_1 and X_2 denote random variables that have the joint p.d.f. $f(x_1, x_2)$ and the marginal probability density functions $f_1(x_1)$ and $f_2(x_2)$ respectively. Let $M(t_1 t_2)$ be the m.g.f. of the distribution. Then prove that X_1 and X_2 are independent if and only if $M(t_1, t_2) = M(t_1, 0)M(0, t_2)$.

18. (a) Let X have a gamma distribution with $\alpha = r/2$, where r is a positive integer and $\beta > 0$. If the random variable $Y = 2X/\beta$ find the p.d.f. of Y.

Or

(b) Prove that if the random variable X is $N(\mu, \sigma^2)$, $\sigma^2 > 0$, then the random variable $V = = (X - \mu)^2/\sigma^2$ is $\chi^2(1)$.

19. (a) Derive 't' distribution.

Or

(b) If F has F–distribution with parameters $r_1 = 5$ and $r_2 = 10$, find a and b so that $\Pr(F \le a) = 0.05$ and $\Pr(F \le b) = 0.95$ and $\Pr(a < F < b) = 0.90$.

20. (a) Prove that (i) $\overline{X}$ is $N(\mu, \sigma^2/n)$ (ii) ns^2/σ^2 is $\chi^2(n-1)$ (iii) $\overline{X}$ and S are independent.

Or

(b) Let $X_1, X_2, \ldots X_n$ be independent random variables having respectively, the normal distributions $N(\mu, \sigma_1^2)$, $N(\mu_2, \sigma_2^2)\ldots$ and $N(\mu_n, \sigma_n^2)$. Then prove that the random variable $Y = k_1X_1 + k_2X_2 + \ldots + k_nX_n$ where $k_1, k_2, \ldots, k_n$ are real constants, be normally distributed with mean $k_1\mu_1 + \ldots + k_n\mu_n$ and variance $k_1^2\sigma_1^2 + \ldots + kn^2\sigma_n^2$.

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(7 pages) **Reg. No. :** ....

Code No. : 5377 **Sub. Code : ZMAM 31**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics – Core

ADVANCED ALGEBRA – I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If V is a vector space of dimension 5 then dim $\dim Hom(Hom(V,F),F)$ is

(a) 125

(b) 10

(c) 25

(d) 5

2. If $U \subseteq W$ are subspaces of a vector space V and if $A(W)$ is the annihilator of W then

(a) $A(U) \subseteq A(W)$

(b) $A(U) \supseteq A(W)$

(c) $A(U) = A(W)$

(d) $A(U)$ and $A(W)$ are not comparable

3. If V is finite-dimensional over F, then $T \in A(V)$ is singular if and only if there exists a

(a) $v \neq 0$ in V such that $vT = 0$

(b) $v \neq 0$ in V such that $vT \neq 0$

(c) $v = 0$ in V such that $vT = 0$

(d) $v = 0$ in V such that $vT \neq 0$

4. If $m(s) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $m(T) = \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix}$ then $m(ST)$ is

(a) $\begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix}$ (b) $\begin{pmatrix} 5 & 6 \\ 5 & 12 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 6 \\ 5 & 12 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 2 \\ 5 & 7 \end{pmatrix}$

5. The matrix M_4 is

(a) $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

6. If M, of dimension 8, is cyclic w.r.t T, then the dimension of MT^3 is

(a) 11 (b) 5

(c) 24 (d) 8^3

7. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 7 & -6 & 3 & 5 \end{pmatrix}$ is the companion matrix of the polynomial

(a) $7 - 6x + 3x^2 + 5x^3 + x^4$

(b) $-7 + 6x - 3x^2 - 5x^3 - x^4$

(c) $-7 + 6x - 3x^2 - 5x^3 + x^4$

(d) $7 - 6x + 3x^2 + 5x^3$

8. If $A = \begin{pmatrix} 1 & 15 & 4 \\ 3 & -1 & 0 \\ 3 & 4 & 5 \end{pmatrix}$ then $tr(A)$ is

(a) 6 (b) 9

(c) 5 (d) 7

9. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ then $(-1)^{\sigma}$ is

(a) 1 (b) −1

(c) σ (d) $-\sigma$

10. $T \in A(V)$ is normal if and only if

(a) $TT^* = TT^*$ (b) $(T^*)^* = T$

(c) $TT^* - T^*T = 0$ (d) $TT^* = 1$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If S, $T \in Hom(V,W)$ and $\lambda \in F$, define $S+T$ and λS. Also show that $S+T \in Hom(V,W)$.

Or

(b) If u, $v \in V$, Prove that $|u,v| \leq \|u\| \cdot \|v\|$.

[P.T.O.]

12. (a) Let A be an algebra with unit element over F and suppose that A is of dimension m over F. Prove that every element in A satisfies some nontrivial polynomial in $F[x]$ of degree at most m.

Or

(b) If V is finite dimensional over F and S, $T \in A(V)$ prove that $r(ST) = r(TS) = r(T)$ for S regular in $A(V)$.

13. (a) Let V_1 be the subspace of V spanned by v, vT, $vT^2, \ldots, vT^{n_1-1}$, where $T \in A(V)$. If $u \in V_1$ is such that $uT^{n_1-k} = 0$ where $0 < k \le n_1$, prove that $u = u_0 T^k$ for some $u_0 \in V_1$.

Or

(b) If $W \subseteq V$ is invariant under T, prove that T induces a linear transformation $\overline{T}$ on V/W defined by $(v + W)\overline{T} = vT + W$.

14. (a) State and prove Jacobson lemma.

Or

(b) Prove that the determinant of a triangular matrix is the product of its entries on the main diagonal.

15. (a) If $T \in A(V)$ is Hermitian, prove that all its characteristic roots are real.

Or

(b) Show that congruence is an equivalence relation.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If V is finite-dimensional, prove that there is an isomorphism ψ of V onto $\hat{\hat{V}}$, the second dual of V.

Or

(b) If V is a finite dimensional inner product space, prove that V has an orthonormal set as a basis.

17. (a) If A is an algebra with unit element over F, prove that A is isomorphic to a subalgebra of $A(V)$ for some vector space V over F.

Or

(b) If $\lambda_1, \lambda_2, \ldots, \lambda_k$ in F are distinct characteristic roots of $T \in A(U)$ and if $v_1, v_2, \ldots v_k$ are characteristic vectors of T belonging to $\lambda_1, \lambda_2 \ldots, \lambda_k$ respectively, then prove that $v_1, v_2, \ldots v_k$ are linearly independent over F.

18. (a) If $T \in A(V)$ has all its characteristic roots in F, prove that there is a basis of V in which the matrix of T is triangular.

Or

(b) "Two nilpotent linear transformation are similar if and only if they have the same invariants" – prove.

19. (a) If the elements S and T in $A(V)$ are similar in $A(V)$, prove that they have the same elementary divisors.

Or

(b) If F is a field of characteristic 0 and if $T \in A(V)$ is such that $trT^i = 0$ for all $i \ge 1$, prove that T is nilpotent.

20. (a) If $T \in A(V)$ is such that $(vT, v) = 0$, for all $v \in V$, prove that $T = 0$. If V is an inner product space over the real field, prove that the above result may be false.

Or

(b) State and prove Sylvester's law.

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Reg. No. : ....

Code No.: 7377 **Sub. Code: ZMAM 31**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023

Third Semester

Mathematics – Core

ADVANCED ALGEBRA – I

(For those who joined in July 2021–2022)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Let U, W be subspaces of V such that $U \subset W$. Let $A(U)$ be the annihilator of U. Then

(a) $A(U) \supset A(W)$

(b) $A(U) \subset A(W)$

(c) $A(U)$ and $A(W)$ are not comparable

(d) $A(U) = A(W)$

2. If $\dim V = 5$, $\dim W = 6$ then $\dim Hom(V,F) + \dim Hom(W,F)$ is

(a) 60 (b) 30

(c) 61 (d) 11

3. The number of operations is an algebra is

(a) 1 (b) 2

(c) 3 (d) 4

4. Let V be two-dimensional over the filed F or real number with a basis v_1, v_2. If T is defined by $v_1T = v_1 + v_2$, $v_2T = v_1 - v_2$, which one of the following is a characteristic root of T

(a) $\sqrt{2}$ (b) 3

(c) 2 (d) 1

5. If M, of dimension 16, is cyclic w.r.t. T, then the dimension of MT^4 is

(a) 64 (b) 4

(c) 12 (d) 20

6. Which of the following is a Jordan block

(a) $\begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$

7. The companion matrix of $-5 + 6x + 7x^2 + x^3 + x^4$ is

(a) $\begin{pmatrix} -5 & 6 & 7 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & -6 & -7 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & 6 & 7 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & 6 & 7 & 0 \end{pmatrix}$

8. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}, C = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$ then

(a) $\det C = \det A + \det B$

(b) $\det C = \det A \bullet \det B$

(c) $\det C = \det A - \det B$

(d) $\det C = 2\det A + \det B$

9. If N is normal and if for $\lambda \in F$, $v(N - \lambda)^k = 0$ then vN is

(a) $\lambda^k v$ (b) λv

(c) $(-\lambda)^k v$ (d) 0

10. If 2,3, –4, –5, –7,0,0 are the characteristic roots of a real symmetric matrix A then the signature of A is

(a) 5 (b) 1

(c) –1 (d) 7

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) If V is finite dimensional, prove that $\psi : V \to \hat{\hat{V}}$ defined by $v\psi = T_v$ for every $v \in V$, where $T_v(f) = f(v)$ for any $f \in \hat{V}$, is an isomorphism of V on $\hat{\hat{V}}$.

Or

(b) If V is a finite dimensional inner product space and if W is a subspace of V, prove that V is the direct such of W and $W^{\perp}$.

12. (a) If V is finite dimensional over F, prove that $T \in A(v)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero

Or

(b) Prove that $\lambda \in F$ is a characteristic root of $T \in A(v)$ if an only if for some $v \neq 0$ is V, $vT = \lambda v$.

13. (a) If V is n-dimensional over F and if $T \in A(v)$ has all its characteristic roots in F, prove that V satisfies a polynomial of degree n over F.

Or

(b) If $T \in A(v)$ is nilpotent, prove that $\alpha_0 + \alpha_1 T + ... + \alpha_m T^m$ where the $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.

14. (a) If V is cyclic relative to T and if the minimal polynomial of T in $F(x)$ is $p(x)$, then prove that for some basis of V the matrix of T is $C(p(x))$.

Or

(b) State and prove Jacobson lemma.

15. (a) If $(vT, vT) = (v,v)$ for all $v \in v$, prove that T is unitary.

Or

(b) If N is normal and $AN = NA$, prove that $AN^* = N^* A$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)
Each answer should not exceed 600 words.

16. (a) Define an inner product space V with an example and show that if $u, v \in V$ then $|(u,v)| \le \|u\| . \|v\|$

Or

(b) Let R be Euclidean ring. Prove that any finitely generated R-module is direct sum of a finite number of cyclic submodules.

17. (a) If V is finite dimensional over F then for S, $T \in A(v)$ prove that $r(ST) \le n(T)$, $r(TS) \le r(T)$ and $r(ST) = r(TS) = r(T)$ for S regular in $A(v)$

Or

(b) Suppose V is the vector space of all polynomials over F of degree 30 less and let D be the differentiation operator : Compute the matrix of D in the basis

(i) $1, x, x^2, x^3$

(ii) $1, 1+x, 1+x^2, 1+x^3$

(iii) If the matrix in part (1) is A and that in part (2) is B, find a matrix C so that $B = cAc^{-1}$.

18. (a) If $T \in A(v)$ has all its characteristic roots in F, prove that there is a basis of V in which the matrix of T is triangular.

Or

(b) Let $T \in A(v)$ and let $p(x) = q_1(x)^{l_1} \, q_2(x)^{l_2} ......q_k(x)^{l_k}$ be the minimal polynomial of T over F.

Let $V_i = \{v \in V | v q_i(T)^{l_i} = 0\}$. For each i = 1,2, k, prove that $V_i \neq (0)$ and $V = V_1 \oplus V_2 + ... \oplus V_k$.

19. (a) If F is a field of characteristic 0 and if $T \in A_F(v)$ is such that $trT^i = 0$ for all $i \ge 1$, prove that T is nilpotent.

Or

(b) Prove that (i) $\det A = \det(A')$ (ii) A is invertible if and only if $\det A \neq 0$.

20. (a) If $T \in A(v)$, prove that (i) $T^* \in A(v)$ (ii) $(T^*)^* = T$ (iii) $(S+T)^* = S^* + T^*$.

Or

(b) State and prove Sylvester's law.

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(6 pages) **Reg. No. : ....**

Code No. : 5378 **Sub. Code : ZMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

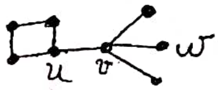
1. Let G be simple. $\varepsilon = \binom{\gamma}{2}$ if and only if G is

(a) A cycle

(b) A path

(c) Complete

(d) A complete bipartite graph

2. Consider the graph G : 

Then $d(u)+d(v)-d(w)$ is

(a) 6 (b) 5

(c) 7 (d) 4

3. Number of spanning trees of the cycle C_4 is

(a) 1 (b) 2

(c) 4 (d) 8

4. The relation connecting K and K' is

(a) $K' \le K$ (b) $K \le K'$

(c) $K + K' = \gamma$ (d) $K + K' = \varepsilon$

5. A connected graph has an Euler trail if and only if it has ———— vertices of odd degree

(a) At most two (b) At least two

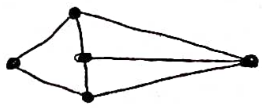
(c) Exactly two (d) No

6. In $C_{1,5}$ the number of edges is

(a) 4 (b) 5

(c) 6 (d) 7

7. The edge chromatic number of the graph



is

(a) 1 (b) 4

(c) 3 (d) 2

8. The value of $r(3, 3)$ is

(a) 6 (b) 9

(c) 3 (d) 0

9. If G is 7-critical then

(a) $\delta = 6$ (b) $\delta \ge 6$

(c) $\delta \ge 7$ (d) $\delta \le 6$

10. If G is a tree, with 4 vertices then $\prod_k(G)$ is

(a) $k(k-1)^3$

(b) k^4

(c) $(k-1)^4$

(d) $k(k-1)(k-2)(k-3)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Draw any graph with 4 vertices and 6 edges. For your graph, write down the incidence matrix.

Or

(b) Define the degree of a vertex and Show that in any graph, the number of vertices of odd degree is even.

12. (a) In a tree, prove that any two vertices are connected by a unique path.

Or

(b) If e is a link of G, prove that

$\tau(G) = \tau(G-e) + \tau(G.e)$

13. (a) For $1 \le m < n/2$, define the graph $C_{m,n}$ with an example and short that $C_{m,n}$ is non Hamiltonian.

Or

(b) Prove that every 3-regular graph without cut edges has a perfect matching.

14. (a) Let $\mathscr{E} = (E_1, E_2, .., E_k)$ be an optimal k-edge colouring of G. If there is a vertex u in G and colours i and j such that i is not represented at u and j is represented at least twice at u, prove that the component of G $[E_i \cup E_j]$ that contains u is an odd cycle.

Or

(b) Define the numbers α and β and show that $\alpha + \beta = \gamma$.

15. (a) In a critical graph, prove that no vertex cut is a clique.

Or

(b) If G is simple, prove that $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$ for any edge e of G.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) Show that, in any group of two or more people, there are always two with exactly the same number of friends inside the group

(ii) Show that if G is simple and $\delta > [r/2] - 1$, then G is connected.

Or

(b) Prove that a graph is bipartite if and only if it contains no odd cycle.

17. (a) Show that any spanning tree $T^x = G[\{e_1, e_2, e_{r-1}\}]$ constructed by Kruskal's algorithm is an optimal tree.

Or

(b) With usual notations, prove that $K \le K' \le \delta$.

18. (a) If G is a simple graph with $\gamma \ge 3$ and $\delta \ge r/2$, prove that G is Hamiltonian.

Or

(b) Prove that G has a perfect matching if and only if $O(G-S) \le (S)$ for all $S \subset V$.

19. (a) State and prove Vizing's theorem.

Or

(b) Prove that $r(k, k) \ge 2^{k/2}$.

20. (a) If G is 4-chromatic, Prove that G contains a subdivision of K4.

Or

(b) If G is a connected simple graph and is neither an odd cycle nor a complete graph, Prove that $\chi \le \Delta$.

(7 pages) **Reg. No. : ....**

Code No. : 5379 **Sub. Code : ZMAM 33**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics – Core

MEASURE AND INTEGRATION

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If Q is the set of rational numbers then the outer measure of Q is

(a) ∞ (b) 0

(c) 1 (d) $\frac{22}{7}$

2. A set of real numbers is said to be a G_δ set provided it is the

(a) intersection of a finite collection of open sets

(b) union of a countable collection of closed sets

(c) intersection of a countable collection of open sets

(d) union of a countable collection of open sets

3. $|f|(x)$ is defined by

(a) $\max\{f(x),0\}$ (b) $\max\{-f(x),0\}$

(c) $\max\{f(x),-f(x)\}$ (d) $f^+(x)-f^-(x)$

4. Consider the statements

(A) Pointwise limit of continuous functions is continuous

(B) Pointwise limit of Riemann integrable functions is Riemann integrable

(a) Both (A) and (B) are true

(b) Neither (A) nor (B) is not true

(c) (A) is true but (B) is not true

(d) (A) is not true but (B) is true

5. If $\phi = 2\chi_{[0,3]} + 5\chi_{[7,9]}$ then $\int_{[0,3]\cup[7,9]} \phi$ is

(a) 16 (b) 7

(c) 5 (d) 0

6. If E_0 is a subset of E of measure zero, then $\int_E f$ is

(a) $\int_{E_0} f$ (b) $\int_{E-E_0} f$

(c) $\int_{E_0-E} f$ (d) zero

7. For an extended real valued function f on E, $f+|f|$ is

(a) $f^+ + f^-$ (b) $2f^+$

(c) $-2f^-$ (d) 0

8. The function $Av_h f$ is defined for all $x \in [a,b]$ and $0 < h \leq 1$ by

(a) $\frac{1}{h}\int_x^{x+h} f$ (b) $\frac{f(x+h)-f(x)}{h}$

(c) $\frac{1}{h}\int_{x-h}^{x+h} f$ (d) $\frac{1}{h}\int_a^b f$

9. Which one of the following is not true?

(a) Linear combinations of absolutely continuous functions are absolutely continuous

(b) Composition of absolutely continuous functions is absolutely continuous

(c) If f is Lipschitz on [a,b] then f is absolutely continuous

(d) Absolutely continuous functions are continuous

10. If $\{A, B\}$ is a Hahn decomposition for γ, then γ^+ is defined by

(a) $\gamma^+(E) = \gamma(E)$ (b) $\gamma^+(E) = \gamma(E \cup A)$

(c) $\gamma^+(E) = \gamma(E \cap A)$ (d) $\gamma^+(E) = -\gamma(E \cap A)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that outer measure is countably sub additive.

Or

(b) Define a measurable set and show that the translate of a measurable set is measurable.

12. (a) Let the function f be defined on a measurable set E. Prove that f is measurable if and only if for each open set, the inverse image of the open set under f is measurable.

Or

(b) Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise $a.e.$ on E to the function f. Prove that f is measurable.

13. (a) Construct a sequence $\{f_n\}$ of Riemann integrable functions whose limit function f is not Riemann integrable.

Or

(b) Let ϕ and Ψ be simple functions defined on a set of finite measure E. For any α and β, prove that $\int_E (\alpha\phi + \beta\Psi) = \alpha\int_E \phi + \beta\int_E \Psi$. Also show that if $\phi \le \Psi$ on E then $\int_E \phi \le \int_E \Psi$.

14. (a) State and prove the monotone convergence theorem.

Or

(b) Let C be a countable subset of (a,b). Prove that there is an increasing function on (a,b) that is continuous only at points in $(a,b) \sim C$.

15. (a) Define an absolutely continuous function. Give an example of a continuous function which is not absolutely continuous.

Or

(b) For an outer measure $\mu^* : 2^x \to [0, \alpha]$, define a measurable set (w.r.t μ^*) and show that the union of two measurable sets is measurable.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that the outer measure of an interval is its length.

Or

(b) Let E be a measurable set of finite outer measure. For each $\varepsilon > 0$, prove that there is a finite disjoint collection of open intervals.

$\{I_h\}_{h=1}^{n}$ for which it $O = \bigcup_{h=1}^{n} I_k$, then $m^*(E \sim O) + m^*(O \sim E) < \varepsilon$.

17. (a) State and prove Egoroff's theorem.

Or

(b) State and prove Lusin's theorem.

18. (a) State and prove the bounded convergence theorem.

Or

(b) Let $\{f_n\}$ be a sequence of nonnegative measurable functions on E. If $\{f_n\} \to f$ pointwise a.e. on E, the prove that $\int_E f \le \liminf \int_E f_n$. Also show that the inequality may be strict.

19. (a) State and prove the Lebesgue dominated convergence theorem.

Or

(b) If the function f is monotone on the open interval (a,b), then prove that it is differentiable almost everywhere on (a,b).

20. (a) Let the function f be continuous on $[a,b]$. If the family of divided difference functions $\{Diff_h f\}_{0<h\le 1}$ is uniformly integrable over $[a,b]$, prove that f is absolutely continuous on $[a,b]$.

Or

(b) Let γ be a singed measure on the measurable space $(X, \mathfrak{M})$. Prove that there is a positive set A for γ and a negative set B for γ for which $X = A \cup B$ and $A \cap B = \phi$

(8 pages) **Reg. No. :** ....

Code No. : 5380 **Sub. Code : ZMAM 34**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Third Semester

Mathematics — Core

TOPOLOGY — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let $X = \{a, b, c, d\}$ which one of the following is a topology on X

   (a) $\{\phi, X, \{a\}, \{c, d\}\}$

   (b) $\{X, \{a\}, \{c\}, \{a, c\}\}$

   (c) $\{\phi, X, \{a, b\}, \{c, d\}\}$

   (d) $\{\phi, \{a\}, \{c, d\}, \{a, c, d\}\}$

2. $[0, 2)$ is

   (a) open in $\mathbb{R}$ but not open in R_l

   (b) open in R_l but not open in $\mathbb{R}$

   (c) open in R_l and open in $\mathbb{R}$

   (d) not open in R_l and not open in $\mathbb{R}$

3. If $f : R \to R$ is defined by $f(n) = 3x + 1$, then $g : R \to R$ is the inverse of f is

   (a) $g(y) = 3y - 1$

   (b) $g(y) = -3y + 1$

   (c) $g(y) = \frac{1}{3}(y - 1)$

   (d) $g(y) = 3(y - 1)$

4. Let $f : A \to X \times Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$ where $f_1 : A \to X$ and $f_2 : A \to Y$ are two functions. If $U \times V$ is a basis element for the product topology of $X \times Y$, then $f^{-1}(U \times V)$ is

   (a) $f_1^{-1}(U) \cap f_2^{-1}(V)$ (b) $f_1^{-1}(U) \cup f_2^{-1}(V)$

   (c) $f_1(U) \cap f_2(V)$ (d) $f_1(U) \cup f_2(V)$

5. In R, define $d(x, y) = |x - y|$. Then $B(3, 0.7)$ is

   (a) (3, 3.7) (b) (2.3, 3.7)

   (c) (– 0.7, 0.7) (d) (3.7, 4.4)

6. The standard bounded metric $\overline{d}$ corresponding to d is defined by

   (a) $\max\{1, d(x, y)\}$

   (b) $\min\{d(x, y), d(y, x)\}$

   (c) $\min\{1, d(x, y)\}$

   (d) $1 + d(x, y)$

7. Let $X = \{a, b, c, d\}$ and let $I = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ then

   (a) $(\{a, b\}, \{c, d\})$ is a separation of X

   (b) $(\{a\}, \{b, c, d\})$ is a separation of X

   (c) $(\{a\}, \{b\})$ is a separation of X

   (d) X has no separation

8. Which one of the following is an open covering for $\mathbb{R}$?

   (a) $\{(n, n+1) / n \in z\}$ (b) $\{(n-1, n) / n \in z\}$

   (c) $\{(n, n+2) / n \in z\}$ (d) $\{[n, n+1] / n \in z\}$

9. A space X is said to be limit point compact is

   (a) every point is a limit point

   (b) every infinite subset of X has a limit point

   (c) every subset of X has a limit point

   (d) every finite subset of X has a limit point

10. The one-point compactification of the real line $\mathbb{R}$ is

    (a) S^1

    (b) S^2

    (c) the extended complex plane

    (d) $\mathbf{C}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies τ and τ' respectively on X. Prove that τ' is fines than τ if and only if for each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subseteq B$.

    Or

    (b) Prove that every finite point set in a Hausdorff space X is closed.

12. (a) State and prove the pasting lemma.

Or

(b) Let $\{X_\alpha\}$ be an indexed family of spaces. Let $A_\alpha \subset X_\alpha$ for each α. If πX_α is given either the product or the box topology, prove that $\pi \overline{A_\alpha} = \overline{\pi A_\alpha}$.

13. (a) Let d and d' be two metrics on the set X, and let τ and τ' be the topologies they induce, respectively. Prove that τ' is fines than τ if and only if for each x in X and each $\varepsilon > 0$, there exists a $\delta > 0$ such that $B_{d'}(x\ \delta) \subset B_d(x, \varepsilon)$.

Or

(b) State and prove the sequence lemma.

14. (a) Prove that the image of a connected space under a continuous map is connected.

Or

(b) Prove that every compact subspace of a Hausdorff space is closed.

15. (a) Give an example of a limit point compact space which is not compact.

Or

(b) Let X be a Hausdorff space. Prove that X is exactly compact if and only if given x in X and given a neighborhood U of x, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subseteq U$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Define a topology. Write down any three topologies on $X = \{a, b, c\}$. Define the finite complement topology on a set X and show that it is a topology on X.

Or

(b) Let X be a topological space. Prove that

(i) ϕ and X are closed

(ii) arbitrary intersections of closed sets are closed

(iii) finite unions of closed sets are closed

17. (a) Let X and Y be topological spaces, let $f : X \to Y$. Prove that the following are equivalent.

(i) For every closed set B of Y, the set $f'(B)$ is closed in X

(ii) f is continuous

(iii) for every subset A of X, one has $f(\overline{A}) \subseteq \overline{f(A)}$.

Or

(b) If each space X_α is a Hausdorff space, prove that πX_α is a Hausdorff space in both the box and product topologies.

18. (a) Define a suitable metric D on on R^ω and show that D induces the product topology on R^ω.

Or

(b) Show that R^ω in the box topology is not metrizable.

19. (a) Prove that a finite Cartesian product of connected space is connected.

Or

(b) Prove that the product of two compact spaces is compact.

20. (a) Define a limit point compact space and a sequentially compact space. If X is metrizable, prove that if X is limit point compact then it is sequentially compact.

Or

(b) If X is a locally compact Hausdorff space that is not itself compact, then prove that X has a one-point compactification Y.

———————

(7 pages) Reg. No. : ....

Code No. : 7383 **Sub. Code : ZMAM 41**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Fourth Semester

Mathematics — Core

ADVANCED ALGEBRA — II

(For those who joined in July 2021-2022)

Time :Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The dimension of the vector space K over the field F is ———— of K over F.
   (a) degree (b) dimension
   (c) basis (d) element

2. A complex number which is not algebraic is called ————
   (a) imaginary (b) real
   (c) transcendental (d) ideal

3. If $f(x) \in F[x]$, a finite extension E of F is called ———— over F if over E but not over any proper subfield of E, $f(x)$ can be factored as a product of linear factors.
   (a) normal (b) extension
   (c) splitting field (d) finite ring

4. The element $a \in K$ is a root of $P(x) \in F[x]$ of ———— if $(x - a)^m \mid p(x)$ whereas $(x - a)^{m+1} \nmid p(x)$
   (a) divisor (b) multiplicity m
   (c) root m (d) basis m

5. If G is a group of automorphisms of K, then the ———— of G is the set of all element $a \in k$ such that $\sigma(a) = a$ for all $a \in G$.
   (a) finite field (b) infinite
   (c) fixed field (d) root

6. The automorphism σ of k is in $G(K, F)$ if and only if $\sigma(\alpha) =$ ———— for every $\alpha \in F$.
   (a) 1 (b) α
   (c) 0 (d) ∞

7. Any two finite fields having the same number of elements are ————
   (a) equal (b) unequal
   (c) constant (d) isomorphic

8. The multiplicative group of nonzero elements of a finite field is ————
   (a) isomorphic (b) fixed
   (c) cyclic (d) unequal

9. For $x = \alpha_0 + \alpha_1 i + \alpha_2 j + \alpha_3 k$ in Q the ———— of x is defined by $x^* = \alpha_0 - \alpha_1 i + \alpha_2 j + \alpha_3 k$
   (a) parallel (b) perpendicular
   (c) adjoint (d) normal

10. If $x \in Q$ then the norm of x is defined by ————
    (a) x (b) xx^*
    (c) x^{-1} (d) $-x$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if a, b in k are algebraic over F, then $a \pm b$, ab, a/b(if $b \neq 0$ re all algebraic over F.

Or

(b) Prove that if $a \in k$ is algebraic of degree n over F, then $[F(a) : F] = n$.

12. (a) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a trivial common factor.

Or

(b) Prove that there is an isomorphism τ^{**} of $\frac{F(x)}{f(x)}$ onto $\frac{F'(t)}{f'(t)}$ with the property that for every

$\alpha \in F \cdot \alpha\tau^{**} = \alpha^1$, $(x + (f(x))\tau^{**} = t + (f'(t))$.

13. (a) Prove that the fixed field of G is a subfield of K.

Or

(b) Let K be a normal extension of F and let H be a subgroup of $G(K, F)$ let $K_H = \{x \in K \mid \sigma(x) = 0$ for all $\sigma \in H\}$ be the fixed field of H. Then prove that $[K : K_H] = O(H)$, and $H = G(K, KH)$.

14. (a) Let D be a division ring of characteristic $P > 0$ with center Z and let $P = \{0, 1, 2 .... (p-1)\}$ be the subfield of Z isomorphic to J_p suppose that $a \in D$, $a \notin Z$ is such that $a^{p^n} = a$ for some $n \geq 1$. Then prove that there exists an $x \notin D$ such that

(i) $x a x^{-1} \neq a$

(ii) $x a x^{-1} \in P(a)$ the field obtained by adjoining a to P.

Or

(b) Let D be a division ring such that for every $a \in D$ there exists a positive integer $n(a) > 1$ such that $a^{n(a)} = a$. Then show that D is a commutative field.

15. (a) If C be the field of complex numbers and suppose that the division ring D is algebraic over C then prove that $D = C$.

Or

(b) State and prove Lagrange identity.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that if L is an algebraic extension of K and if K is an algebraic extension of F then L is an algebraic extension of F.

Or

(b) Show that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

17. (a) Prove that any splitting fields E and E' of the polynomials $f(x) \in F[x]$ and $f'(t) \in F'[t]$ respectively are isomorphic by an isomorphism ϕ with the property that $\alpha\phi = \alpha'$ for every $\alpha \in F$.

Or

(b) Prove that if F is of characteristic O and if a, b are algebraic over F, then there exists an element $C \in F(a, b)$ such that $F(a, b) = F(c)$.

18. (a) Prove that if K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

Or

(b) If K is a finite extension of F, then prove that $G(K, F)$ is a finite group and its order $O(G(K, F))$ satisfies $O(G, (K, F)) \leq [K : F]$.

19. (a) Prove that a finite division ring is necessarily a commutative field.

Or

(b) Let G be a finite abelian group enjoying the property that the relating $x^n = e$ is satisfied by atmost n element of G for every integer n. Then prove that G is a cyclic group.

20. (a) Prove that every positive integer can be expressed as the sum of squares of four integers.

Or

(b) State and prove Frobenius theorem.

(7 pages) Reg. No. : ....

Code No. : 5384 **Sub. Code : ZMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Fourth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A function u is harmonic if it satisfies

(a) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$ (b) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0$

(c) $\frac{\partial^2 u}{\partial x^2}=-\frac{\partial^2 u}{\partial y^2}$ (d) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}, \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

2. A rational function $R(z)$ of order p has ———— zeros and ———— poles

(a) $p, p-1$ (b) $p-1, p$

(c) p, p (d) $p, p+1$

3. If $w=s(z)=\frac{az+b}{cz+d}$, $ad-bc\neq 0$, then $s^{-1}(w)$ is given by

(a) $\frac{b-dw}{-cw+a}$ (b) $\frac{dw-b}{-cw+a}$

(c) $\frac{dw-b}{a-cw}$ (d) $\frac{cz+d}{az+b}$

4. (z_1,z_2,z_3,z_4) is the image of z_1 under the linear transformation which carries z_2,z_3,z_4 into

(a) $0, 1, \infty$ (b) $1, \infty, 0$

(c) $1, 0, \infty$ (d) $1, 1, 1$

5. If $\int_\gamma f(z)dz=5+i$, then $-\int_\gamma f(z)dz$ is

(a) 0 (b) $5+i$

(c) $5-i$ (d) $\sqrt{26}$

6. When C is a circle about G, then $\int_C \frac{dz}{z-a}$ is

(a) 0 (b) 2π

(c) $2\pi i$ (d) $2\pi a i$

7. If $n(\gamma,a)=5$ then $n(-\gamma,a)-n(\gamma,-a)$ is

(a) -10 (b) 5

(c) 10 (d) 0

8. The value of $\int_{|z|=1}\frac{e^z}{z}dz$ is

(a) 2π (b) $2\pi i$

(c) 0 (d) ∞

9. The residue of $\frac{e^z}{(z-a)^2}$ at $z=a$ is

(a) e^a (b) ∞

(c) $\frac{e^a}{z-a}$ (d) 1

10. If f has a pole or order h, then $f\frac{1}{f}$ has the radius

(a) h (b) $-h$

(c) 0 (d) ∞

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Derive Cauchy-Riemann differential equation for any analytic function.

Or

(b) Prove that $\sum a_n z^n$ and $\sum n a_n z^{n-1}$ have the same radius of convergence.

12. (a) Explain a conformal mapping.

Or

(b) Prove that the reflection $z \to \bar{z}$ is not a linear transformation.

13. (a) Prove that the time integral $\int_\gamma Pdx+qdy$ defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x,y)$ in Ω such that $\frac{\partial U}{\partial x}=p$, $\frac{\partial U}{\partial y}=q$.

Or

(b) Compute $\int_{|z|=r} x\,dz$ for the positive sense of the circle.

14. (a) State and prove Morera's theorem.

Or

(b) State and prove the fundamental theorem of algebra.

15. (a) State and prove the residue theorem.

Or

(b) Compute $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}; a > 1.$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If all zeros of a polynomial $p(z)$ lies in a half plane, prove that all zeros of the derivative $p'(z)$ lie in the same half plane.

Or

(b) Find the radius of convergence of the power series

(i) $\sum n^p z^n$

(ii) $\sum n! z^n$

17. (a) If $T_1 z = \frac{z+2}{z+3}$, $T_2 z = \frac{z}{z+1}$, find $T_1 T_2 z$, $T_2 T_1 z$ and $T_1^{-1} T_2 z$.

Or

(b) Investigate the geometric significance of symmetry when

(i) C is a straight line

(ii) C is a circle of center a and radius R.

18. (a) If the function $f(z)$ is analytic on a reactance R, prove that $\int_{\partial R} f(z)dz = 0$.

Or

(b) If $f(z)$ is analytic in an open disc Δ, prove that $\int_{\gamma} f(z)dz = 0$ for every closed curve γ in Δ.

19. (a) With usual notation prove that $F_n'(z) = nF_{n+1}(z)$ if $F_n(z) = \int_{\gamma} \frac{\phi(\xi)\,d\xi}{(\xi - z)n}$.

Or

(b) State and prove Weierstrass theorem for an essential singularity.

20. (a) State and prove Roucher's theorem.

Or

(b) Evaluate $\int_{-\infty}^{+\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

———

(8 pages) Reg. No. : ....

Code No. : 5385 **Sub. Code : ZMAM 43**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Fourth Semester

Mathematics – Core

FUNCTIONAL ANALYSIS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which one of the following is not true?
   (a) Every complete normed linear space is a Banach space
   (b) The norm is a continuous function
   (c) If the linear transformation T is continuous then T is bounded
   (d) If M is a closed linear subspace of a normed linear space N, then N/M is a Banach space.

2. Let N and N' be normed linear spaces. An isometric isomorphism of N into N' is a one – to – one linear transformation T of N into N^1 such that
   (a) $\|T(x)\| \le \|x\|$ for every x in N
   (b) $\|T(x)\| = \|x\|$ for every x in N
   (c) $\|T(x)\| = 1$
   (d) $T(x) = T(y) \Rightarrow x = y$

3. If X is a compact Hausdorff space, then $\mathcal{C}(X)$ is reflexive if and only if
   (a) X is a finite set
   (b) X is a countable set
   (c) X is complete
   (d) X is a Banach space

4. Let T be a linear transformation of B into B'. The graph of T is a subset of
   (a) B (b) B'
   (c) $B \times B'$ (d) $B' \times B$

5. The parallelogram law states that
   (a) $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 - 2\|y\|^2$
   (b) $\|x+y\|^2 - 2\|x\|^2 = 2\|y\|^2 - \|x-y\|^2$
   (c) $\|x+y\|^2 - \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$
   (d) $\|x+y\|^2 + 2\|x\|^2 = \|x-y\|^2 + 2\|y\|^2$

6. If S is a non-empty subset of a Hilbert space then $S^{\perp\perp\perp}$ is
   (a) S (b) $S^{\perp}$
   (c) ϕ (d) $S^{\perp\perp}$

7. Which one of the property is not the property of the adjoint operation $T \to T^*$ on $\mathcal{B}(H)$
   (a) $(T_1 + T_2)^* = T_2^* + T_1^*$
   (b) $(T_1 T_2)^* = T_1^* T_2^*$
   (c) $T^{**} = T$
   (d) $\|T^* T\| = \|T\|^2$

8. A self – adjoint operator A is said to be positive if
   (a) $(Ax, Ax) \ge 0$ for all x
   (b) (Ax, x) is real for all x
   (c) $(Ax, x) \ge 0$ for all x
   (d) $\|A^2\| = \|A\|^2$

9. An operator T on H is self adjoint if and only if
   (a) $T + T^* = 0$
   (b) (Tx, x) is real for all x
   (c) $TT^* = T^*T$
   (d) $\|T^* x\| = \|Tx\|$ for every x

10. If P is the projection on M then $I - P$ is the projection on
    (a) M
    (b) $M^{\perp}$
    (c) $I - M$
    (d) $M - M$

[P.T.O.]

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define a Banach space with two examples.

Or

(b) If M is a closed linear subspace of a normed linear space N and x_0 is vector not in M, prove that there exists a functional f_0 in N^* such that $f_0(m) = 0$ and $f_0(x_0) \neq 0$.

12. (a) If N is a normed linear space, prove that the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak* topology.

Or

(b) State and prove the closed graph theorem.

13. (a) Prove that the inner product in a Hilbert space is jointly continuous.

Or

(b) Let M be a closed linear subspace of a Hilbert space H, let x be a vector not in M, and let d be the distance from x to M. Prove that there exists a unique vector y_0 in M such that $\|x - y_0\| = d$.

14. (a) Let $\{e_1, e_2, \ldots, e_n\}$ be a finite orthonormal set in a Hilbert space H. If x is any vector in H, prove that $\sum_{i=1}^{n} |(x, e_i)|^2 \leq \|x\|^2$ and $x - \sum_{1}^{n} (x, e_i)e_i \perp e_j$ for each j.

Or

(b) Show that an orthonormal set in a Hilbert space is linearly independent.

15. (a) If T is an operator on H for which $(Tx, x) = 0$ for all x, prove that $T = 0$.

Or

(b) Prove that an operator T on H is unitary $\Leftrightarrow$ it is an isometric isomorphism of H onto itself.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Let M be a closed linear subspace of a normed linear space N. Prove that N/M is a normed linear space. If N is a Banach space, prove that N/M is also a Banach space.

Or

(b) If N' is a Banach space, prove that the normed linear space (N, N') is also a Banach space.

17. (a) Define a reflexive space with an example. If B is a Banach space, prove that B is reflexive $\Leftrightarrow B^*$ is reflexive.

Or

(b) Let B and B' be Banach spaces. Let T be a continuous linear transformation of B onto B'. Prove that the image of each open sphere centered on the origin in B contains an open sphere centered on the origin in B'.

18. (a) State and prove the Banach – Steinhaus theorem.

Or

(b) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

19. (a) State and prove Bessel's inequality.

Or

(b) Let H be a Hilbert space and let f be an arbitrary functional is H^*. Prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H.

20. (a) If T is an operator on H, prove that T is normal $\Leftrightarrow$ its real and imaginary parts commute

Or

(b) If P is a projection on H with range M and null space N, prove that $M \perp N \Leftrightarrow P$ is self adjoint. Also prove that $N = M^{\perp}$ in this case.

(8 pages) Reg. No. : ....

Code No. : 5386 **Sub. Code : ZMAM 44**

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2023.

Fourth Semester

Mathematics – Core

TOPOLOGY – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. A space which contains a countable dense subset is called

   (a) Separable

   (b) Lindelöf

   (c) Second countable

   (d) Compact

2. Another name for Regular space is

   (a) T_4

   (b) $T_{2\frac{1}{2}}$

   (c) $T_{3\frac{1}{2}}$

   (d) T_3

3. Every regular Lindeloff space is

   (a) normal

   (b) completely regular but not normal

   (c) regular but not completely regular

   (d) compact and Hausdorff.

4. A space X is completely regular then it is homeomorphic to a subspace of

   (a) $[0, 1]^J$

   (b) $\mathbb{R}^n$ where n is a finite

   (c) $\mathbb{R}^J$

   (d) $(0, 1)^J$ where J is uncountable

5. Tietze extension theorem implies

   (a) The Urysohn Metrization theorem

   (b) Heine- Borel Theorem

   (c) The Urysohn lemma

   (d) The Tychonof theorem.

6. Find the correct answer

   (a) Subspace of a Normal space is normal

   (b) Product of Normal spaces is normal

   (c) R_ℓ^2 is completely regular

   (d) R_K is regular but not normal

7. The set ———————— is locally finite in R?

   (a) $\{(n-1, x+1) : n \in Z\}$

   (b) $\left\{\left(0, \frac{1}{n}\right) : n \in Z_+\right\}$

   (c) $\left\{\left(\frac{1}{n+1}, \frac{1}{n}\right) : n \in Z_+\right\}$

   (d) $\{(x, x+1) : x \in R\}$

8. Let $\mathcal{A} = \{(n-1, n+1) : n \in Z\}$. Which of the following refine $\mathcal{A}$.

   (a) $\left\{\left(n-\frac{1}{2}, n+\frac{3}{2}\right) : n \in Z_+\right\}$

   (b) $\left\{\left(n+\frac{1}{2}, n+\frac{3}{2}\right) : n \in Z_+\right\}$

   (c) $\left\{\left(n-\frac{1}{2}, n+2\right) : n \in Z_+\right\}$

   (d) $\{(x, x+1) : x \in R\}$

9. Which of the following is not true

   (a) Every non empty open subset of the set of irrational numbers is of first category

   (b) Open subspace of a Baire space is a Baire space

   (c) Rationals as a subspace of real numbers is not a Baire space.

   (d) If $X = \bigcup_{n=1}^{\infty} B_n$ and X is a Baire space with $B_1 \neq \phi$, then atleast one of $\overline{B_n}$ has nonempty interior.

10. Find the incorrect statement

    (a) Any set X with discrete topology is a Baire space

    (b) Every locally compact space is a Baire space

    (c) [0, 1] is a Baire space

    (d) The set of irrationals is not a Baire space

[P.T.O.]

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a space with one point sets in X are closed. Prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$.

Or

(b) Define $\mathbb{R}_k$ topological space. Prove that $\mathbb{R}_k$ is Hausdorff but not regular.

12. (a) Examine the proof of Urysohn lemma and show that for a given r, $f^{-1}(r) = \left(\bigcap_{p>r} U_p - \bigcup_{q<r} U_q \right)$, where p and q are rational.

Or

(b) Prove that every normal space is completely regular and completely regular space is regular.

13. (a) State and prove imbedding theorem.

Or

(b) Prove that Urysohn lemma can be proved by using Tietze extension theorem.

14. (a) Let A be a locally finite collection of subsets of X. Then prove that (i) The collection $B = \{\overline{A} : A \in \mathcal{A}\}$ is locally finite. (ii) $\overline{\bigcup_{A \in \mathcal{A}} A} = \bigcup_{A \in \mathcal{A}} \overline{A}$.

Or

(b) Define finite intersection property. Let X be a set and D be the set of all subsets of X that is maximal with respect to finite intersection property. Show that (i) $x \in \overline{A} \forall A \in D$ if and only if every neighborhood of x belongs to D. (ii) Let $A \in D$. Then prove that $B \supset A \Rightarrow B \in D$.

15. (a) Define a first category space. Prove that X is a Baire space if and only if 'given any countable collection $\{U_n\}$ of open sets in X, U_n is dense in $X \forall n$, then $\cap U_n$ is also dense'.

Or

(b) Define a Baire space. Whether Q the set of rationals as a space is a Baire space? What about if we consider Q as a subspace of real numbers space. Justify your answer.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) What are the countability axioms. Prove that the space $\mathbb{R}_L$ satisfies all the countability axioms but the second.

Or

(b) Prove that product of Lindelof spaces need not be Lindelof.

17. (a) Define a regular space, a Lineloff space and a normal space. Prove that every regular Lindeloff space is normal.

Or

(b) (i) Prove that every normal space is completely regular and completely regular space is regular.

(ii) Prove that product of completely regular spaces is completely regular.

18. (a) State and prove Tietze extension theorem.

Or

(b) State and prove Uryzohn's metrization theorem.

19. (a) Let X be a metrizable space. If A is an open covering of X, then prove that there is an open covering ξ of X refining A that is countably locally finite.

Or

(b) State and prove Tychonoff theorem.

20. (a) Let X be a space; let (Y, d) be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions such that $f_n(x) \to f(x)$ for all $x \in X$, where $f : X \to Y$. If X is a Baire space, prove that the set of points at which f is continuous is dense in X.

Or

(b) State and prove Baire Category theorem.

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